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# HITEMP Material and Structural Optimization Technology Transfer

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# **HITEMP Material and Structural Optimization Technology Transfer**

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## **Abstract**

The feasibility of adding viscoelasticity and the Generalized Method of Cells (GMC) for micro-mechanical viscoelastic behavior into the commercial HyperSizer structural analysis and optimization code was investigated. The viscoelasticity methodology was developed in four steps. First, a simplified algorithm was devised to test the iterative time stepping method for simple 1-D multiple ply structures. Second, GMC code was made into a callable subroutine and incorporated into the 1-D code to test the accuracy and usability of the code. Third, the viscoelastic time-stepping and iterative scheme was incorporated into HyperSizer for homogeneous, isotropic viscoelastic materials. Finally, the GMC was included in a version of HyperSizer. MS Windows executable files implementing each of these steps is delivered with this report, as well as source code. The findings of this research are that both viscoelasticity and GMC are feasible and valuable additions to HyperSizer and that the door is open for more advanced non-linear capability, such as viscoplasticity.

## **Summary**

During the course of this work, the feasibility of including viscoelasticity and the Generalized Method of Cells (GMC) into a commercial version of HyperSizer was demonstrated through two simplified codes and a special research version of HyperSizer. There were no prohibitive method, analytical, or software roadblocks to implementing viscoelasticity and GMC into the commercial version of HyperSizer. Further, we see a potential for future research that could extend the present methodologies to viscoplasticity or non-linear post buckling analyses.

When the process of incorporating the GMC was first begun, the GMC code was setup in such a way as to store large matrices within the routine for each analyzed ply. This led to very large memory requirements (40 MB per ply). This problem was resolved by the subcontractor, Dr Orozco, by solving each of these matrices on the fly and not storing them. The final version was able to solve for 100 plies using only 23 megabytes of memory. In addition to problems with memory, we were concerned with the amount of time that the process would take as a large matrix inversion was being done every time a new ply was sent to the GMC routine. This problem was solved using a sub-iteration technique that allowed single plies to be analyzed multiple times and prevent the matrix inversion from occurring for each call to the GMC routine.

## **Demonstrated Technologies**

*First*, with the development of the Generalized Method of Cells (GMC) code spanning most of 1997, an algorithm was designed to implement this technology into the commercially available code called HyperSizer. The algorithm, outlined in the attachment, "Progress on Laminate/Panel Viscoelastic Integration," was applied first to a simplified problem involving viscoelasticity in 1-D material plies with homogeneous isothermal viscoelastic and elastic materials. This simplified algorithm was developed into a standalone MS Windows (95/98/NT) executable called **Multiplies**. This program is delivered with this report as a Microsoft Visual Basic source code and Windows executable file. **Multiplies** accomplished several things:

- The feasibility of the iterative, time stepping algorithms depicted in the in the attachment, "Progress on Laminate/Panel Viscoelastic Integration."
- The feasibility of a method devised by Collier Research Corporation and the subcontractor, University of Virginia to increase the efficiency of the basic forward-Euler time stepping scheme. This method involves doing sub-iterations at each time step to increase the accuracy without adversely affecting the total run time.
- In the course of development, several closed form benchmark solutions were developed against which some of the later HyperSizer generated results could be compared.

*Second*, the GMC FORTRAN code delivered by the sub-contractor was distilled into a callable subroutine for inclusion into the HyperSizer main program. Before incorporating this code into HyperSizer, the callable routine, called **EpsEtaGMC** was incorporated into the simplified code, **Multiplies** to arrive at a new code called **MultipliesGMC**. This

code, also a Visual Basic source code with Windows executable is delivered with this final report. **MultipliesGMC** demonstrated the following:

- The feasibility of the iterative, time-stepping algorithm with sub-iteration technique applied to the simplified viscoelastic problem using the UVA. generated GMC method
- The usability of the GMC callable subroutine for a separate program.
- When compared with the closed form solutions and the solutions generated with **Multiplies**, the accuracy of the GMC methodology was demonstrated, at least for homogeneous material problems.

*Third*, the HyperSizer program was modified to add a viscoelastic capability using the same algorithm as in **Multiplies**, albeit a 3-D version. The first tests were performed without GMC to simplify the procedure. The major task involved here was to add a time-stepping ability. As HyperSizer normally is restricted to doing point-analyses, it previously had no ability to step through time. In the course of this implementation, attention must be paid to the histories of the viscoelastic plies. Because HyperSizer analysis is based on analysis objects, and each object carries unique strain fields and is composed of many plies, the stresses, strains and viscous stresses and strains is tracked for every ply as part of every analysis object. To put this in perspective, if a complicated panel concept, such as a hat-stiffened panel is composed of composite panels with, say, 18 plies for the facesheet and six plies for the hats, the number of separate plies tracked as part of the analysis is over 60. That is, 60 plies must be calculated for every panel. This analysis capability demonstrates that:

- The iterative time-stepping algorithm with sub-iteration technique is feasible in a HyperSizer full panel implementation.
- Again using closed-form benchmarks, the HyperSizer implementation of the Viscoelastic solution demonstrates consistent and stable results

*Finally*, the HyperSizer program with viscoelastic capability was modified to include the same callable subroutine module **EpsEtaGMC** that implemented the UVA GMC FORTRAN code. This modification was relatively minor based on the earlier work performed to implement viscoelasticity into the main HyperSizer code and the testing and research that went into building and testing the GMC routine through **MultipliesGMC**. This final code, **Hs\_Visco**, is delivered with this final report as a MS Windows Executable. The GMC FORTRAN subroutine, **EpsEtaGMC** and two other supporting routines are included as electronic files. This code really demonstrates the promise of including the GMC methodology into the HyperSizer commercial code. Although the code delivered with the final report is still considered to be 'research' code, it does tell us that there are no roadblocks to implementing viscoelasticity and GMC into the commercial version of HyperSizer.

### ***Commercializing HyperSizer Viscoelasticity***

Implementing viscoelasticity in HyperSizer proved to be valuable even without the GMC methodology. It is feasible to solve problems in which only certain parts of a complex panel would be viscoelastic in nature. For example, for a panel only the facesheet might go viscoelastic while the stiffeners remained elastic. The consistent stiffness

formulations of HyperSizer ensure that during this process, the forces would remain in balance over the entire panel. At the present time, there are two shortcomings in the viscoelastic formulation that would have to be addressed for commercialization.

First, the current implementation of the viscoelastic parameters requires that each ply of each analysis object be entered as a separate material. While the histories of each of these plies would certainly need to be kept, there is no reason that multiple plies could not point to a single material. As implemented right now, the material for each ply is entered through a manually generated ASCII data file and the amount of data that must be generated for a complex panel concept is prohibitive. For an entire vehicle with many individual panels, the amount of data that would have to be generated for many panels would be impractical. If multiple plies pointed at a single material, the amount of data needed would be more manageable.

Second, in order to be commercially viable, the viscoelastic capability would have to be integrated with the current database/GUI scheme. This would require generation of viscoelastic material data to be integrated into the existing material editor forms and time-step/total time/sub-iteration control to be added to the existing project editor forms. In addition to integrating the viscoelastic parameters into the interface, support would have to be added to allow for material properties that vary with temperature. Currently all viscoelasticity in HyperSizer is isothermal.

### ***Commercializing HyperSizer Viscoelasticity with GMC***

The Generalized Method of Cells methodology created by the sub-contractor has proven to be valuable in its ability to model the micro-mechanical behavior of any general uni-directional composite material. The commercialization potential for this capability exists but is a little farther away. There are several things that must be accomplished with the GMC to make it a part of the commercial HyperSizer code.

First, just as with the homogeneous viscoelastic algorithm, the material properties for the current implementation of the GMC must be entered for every ply of every analysis object. This information must be simplified such that multiple plies can point at the same material and the materials data must be integrated into the HyperSizer database/graphical user interface scheme. This is complicated by the fact that the current GMC subroutine reads in its own data file (separate from HyperSizer) and keeps track of all of its own data in the course of an analysis. This is inconsistent with the way data structures in HyperSizer, which are database driven, are handled. Therefore, the input of all material data must be transferred from the GMC routine to the HyperSizer GUI. In addition, the GMC routine opens several output data files and writes data into them. Again, this is inconsistent with the HyperSizer implementation and the GMC must be modified to give all input and output control to the HyperSizer calling program.

Second, in addition to just the FileIO, it would be preferable if the control of the history data and all allocable arrays were turned over to HyperSizer. Presently, the GMC routine, while having a clean interface with HyperSizer, keeps all of its history and ply

data separately from HyperSizer. It would be more efficient and cleaner (and easier to modify) if the data structures were kept track of by HyperSizer and passed to the GMC routine as needed.

Third, the GMC routine is currently limited to 100 plies. This means that the code would only handle one or two panels at the same time and is the biggest challenge that would have to be overcome to make the code truly commercializable as part of HyperSizer. One of the strengths of HyperSizer is the ability to model a complex structure, such as a complete airframe, very quickly. The GMC limitation of 100 plies would be a major limitation of this ability and must be overcome for successful commercialization.

### ***Overall Commercialization Potential***

We see the commercialization potential of the viscoelastic and GMC capability to be high. As discussed above, this would require additional effort on our end in the integration and interface to make the capability more tightly coupled with the way HyperSizer works. In addition, to make the GMC commercializable would require effort on the part of the sub-contractor to make the GMC routine fit more tightly into the HyperSizer database schema by turning over control of the material and history data to HyperSizer and increasing the number of plies. Other than these implementation issues, there were no major roadblocks found to adding a commercializable viscoelastic capability into HyperSizer.

### ***Feasibility***

The effort of this contract has definitely demonstrated the potential for using viscoelasticity and GMC in HyperSizer. The algorithms developed and implemented for viscoelasticity in HyperSizer have demonstrated that the coarse-mesh geometry modeling techniques used by HyperSizer are compatible with both viscoelasticity and the GMC method.

This research has also demonstrated that HyperSizer is compatible without limitation with non-linear, iterative, and time stepping schemes such as viscoelasticity. Using similar techniques there are not roadblocks for extending this capability beyond the current viscoelastic methodology into **viscoplasticity**. In addition to viscoplasticity, other non-linear methods could also be implemented using these techniques. For example, there has recently been interest by HyperSizer users in post-buckling strength analysis for panels and beams. We see the present research as providing a powerful base from which to launch such capabilities.

### **Viscoelasticity Implementation in HyperSizer**

Using the same time-stepping algorithm as outlined in the attachment, "Progress on Laminate/Panel Viscoelastic Integration," the capability to perform viscoelastic calculations on individual plies was added to HyperSizer. HyperSizer uses the concept of 'analysis objects' to carry out its thermo-mechanical failure and stress analyses. An analysis object might be the clear span between hat stiffeners for a hat stiffened panel, the web of a hat stiffener, the hat crown, the closed span of the facesheet over the hat, or the 'combo' object composed of the flange and facesheet combined. For a composite material, each of these analysis objects is made up of multiple plies, and for the viscoelastic analysis, the history of each ply in each analysis object must be kept track of. For the homogeneous viscoelastic material analysis a data structure and pointer system was created in HyperSizer to hold this history and implement a forward Euler time-stepping scheme.

In order to use this methodology, a separate MS Windows executable file is included with this report called **Hs\_Visco.exe**. This file is located in the Program Files\HyperSizer\Executable directory. (Note: This file is not part of the normal HyperSizer installation). The procedure for setting up a viscoelastic run in HyperSizer is very similar to setting up a regular HyperSizer run. First, the user sets up a normal HyperSizer analysis run (see the HyperSizer documentation for details) and runs the normal HyperSizer analysis module to initialize the analysis. Next, a file is created in the project materials directory called \*.VMT (Viscoelastic MaTerial) which has viscoelastic properties for every ply in the analysis. To date, since these files are being completed by hand, only simple single panel analyses have been performed. An example \*.VMT file for a three-ply analysis is listed here:

```
8000.      Total Time (s)
200.       Time Step size (s)
1          Number of sub-iterations
0          GMC analysis flag ( 1 = GMC; 0 = no GMC)
Ply Number (Hot Epoxy)
  1
Damping Coefficient; ViscoElastic Stiffness; Elastic Stiffness
58013050., 446701., 48586.
Ply Number (Cold Epoxy)
  2
Damping Coefficient; ViscoElastic Stiffness; Elastic Stiffness
21319797., 588397., 28861.
Ply Number (Hot Epoxy)
  3
Damping Coefficient; ViscoElastic Stiffness; Elastic Stiffness
58013050., 446701., 48586.
ENDFILE
```

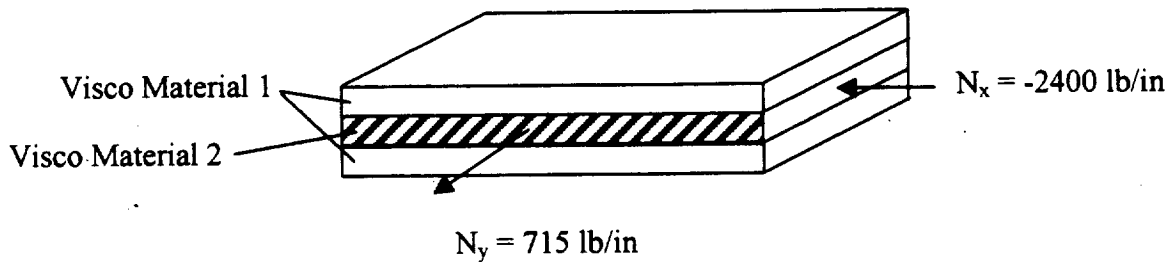
The beginning of the file contains the viscoelastic analysis control parameters, total analysis time, time step, the number of sub-iterations to perform per time step and a control over whether the GMC is to be performed. For each ply to be analyzed, the



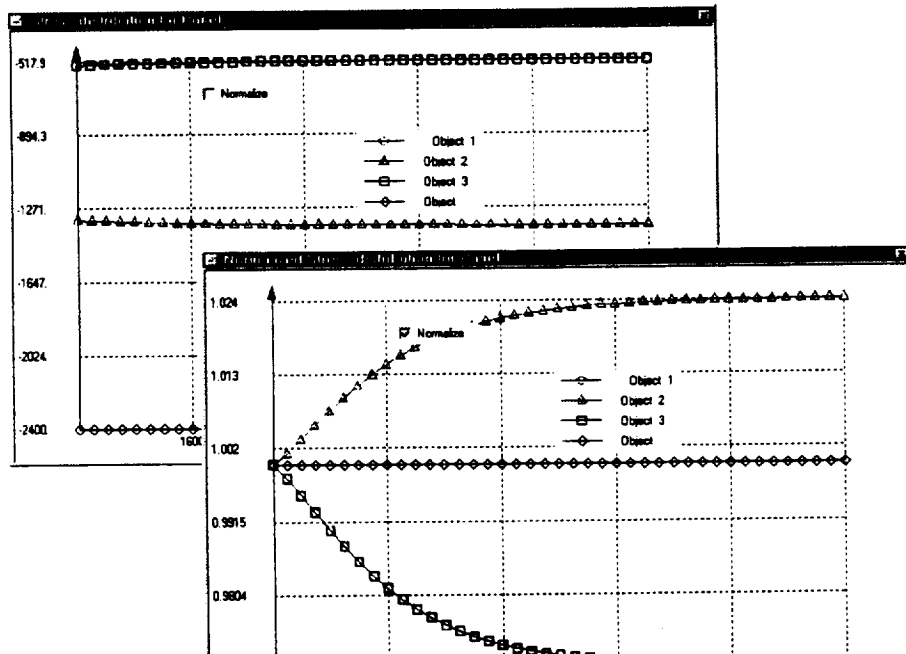
damping coefficient,  $\eta$  ( $\text{psi}\cdot\text{s}^{-1}$ ), the elastic stiffness coefficient,  $E_s$  (psi) and the viscous stiffness coefficient,  $E_m$  (psi). Note that these stiffnesses override the stiffnesses that are specified in the HyperSizer Graphical User interface. This is a disconnect in the integration of the viscoelasticity into HyperSizer and will be resolved in any follow-on development. Once this file is setup, simply go to the Project Input directory and drag the Project.ini file onto the executable, Hs\_Visco to run the viscoelastic analysis. At the end of analysis, an object based plot of stress distribution will automatically pop up to indicate the stress distribution among the various plies.

## Results

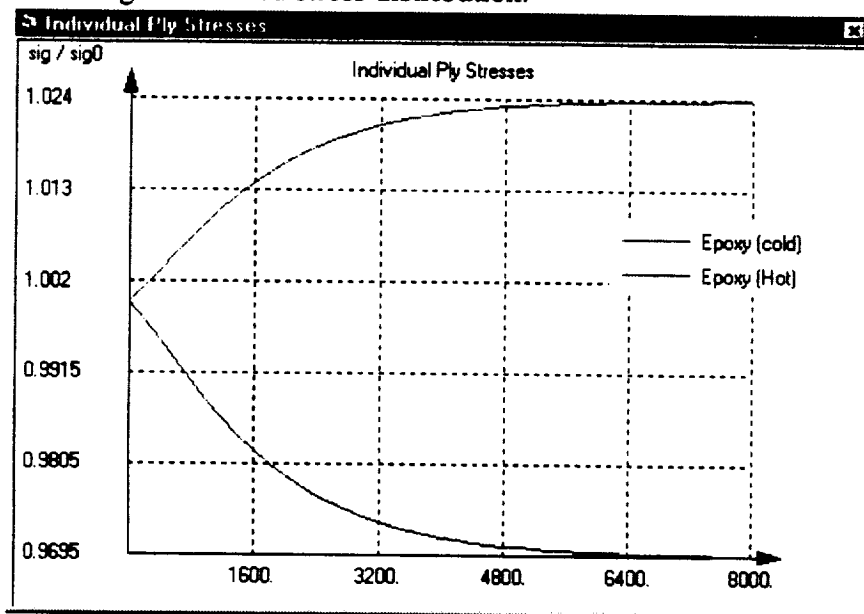
This code was used to solve a simple problem involving an unstiffened plate composed of three homogeneous viscoelastic plies with uniform compression loading. This is implemented as a simple panel concept from the unstiffened plate/sandwich family with the \*.VMT file from the previous page. In order to compare the results of this analysis to those presented in the attached report, "Progress on Laminate/Panel ViscoElastic Integration," a transverse load,  $N_y = -v \cdot N_x$  was applied so that the response of the panel would be solely in the X (longitudinal) direction. This was done using the user-defined load capability in HyperSizer.



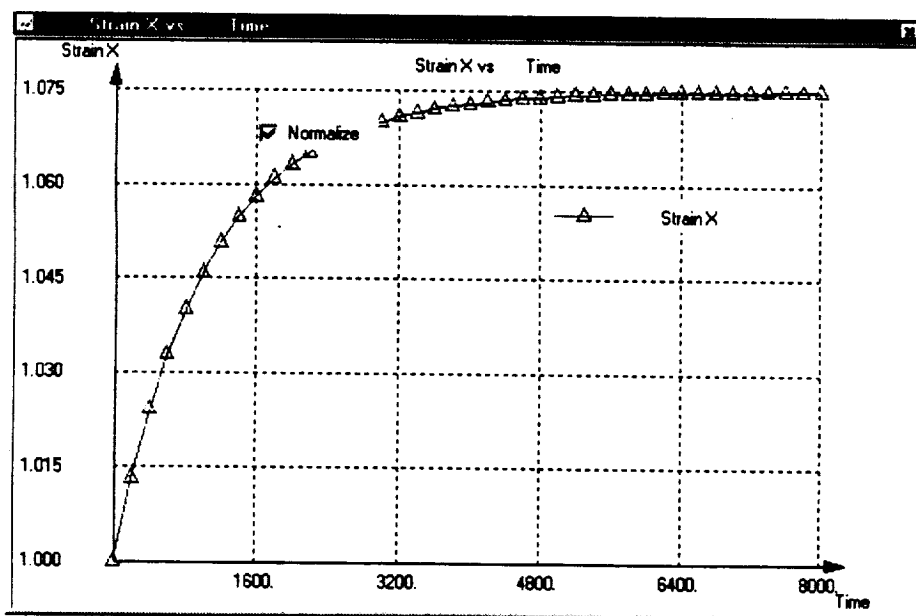
Running the analysis results in the following plot that shows the amount of force in each of the three objects:



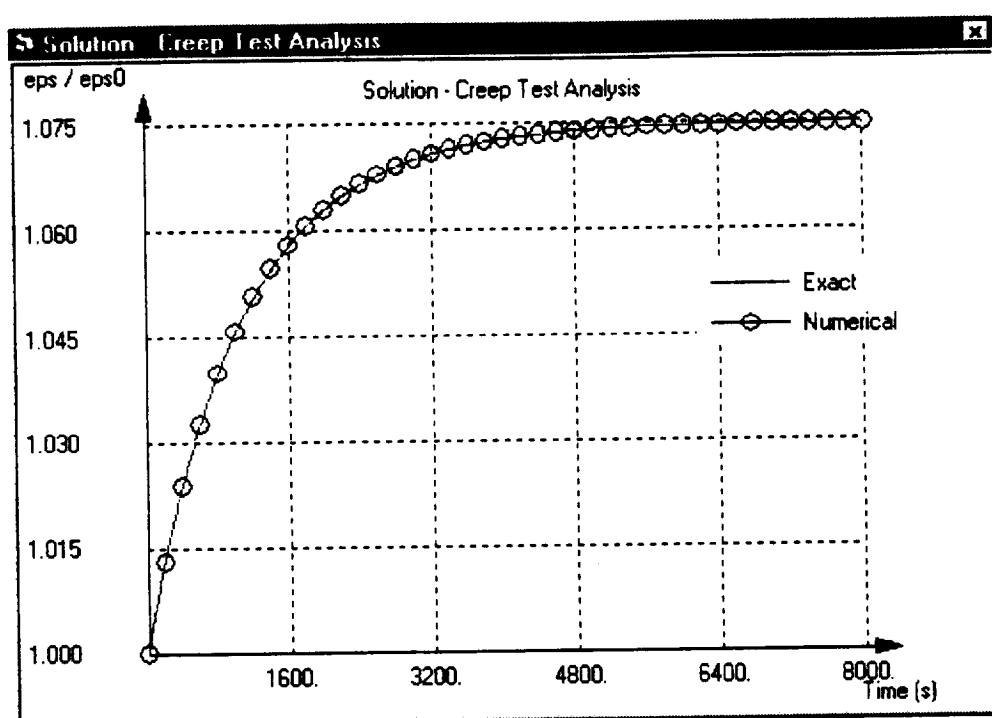
The example performed here has two viscoelastic materials. The center ply, which is less viscoelastic (i.e.  $E_m$  is lower) is twice as thick as the outer plies. This makes sure that the stack is symmetric and that the stack will not experience any bending loads during the analysis. As the figure illustrates, during the analysis, creep occurs and the less viscoelastic center ply picks up more of the total force while the outer plies pick up less force. The summed force (indicated as "Object") remains constant as it should in an analysis of creep. The same case run with the standalone code, **Multiplies** results in the following normalized stress distribution:



Note that this matches almost exactly with the plot given for the HyperSizer analysis. Since this is a creep test, it is useful to plot the solution for normalized strain over the course of the analysis.



These results match very closely with the normalized strains achieved with the **Multiplies** code:



The results obtained with this non-GMC implementation of viscoelasticity demonstrate that the capability has been successfully integrated into HyperSizer, at least for very simple geometry problems. The methodology has been demonstrated for more complex panel shapes such as the integral blade stiffened panel. For more complex shapes, while there are no exact solutions against which to compare, it is worth noting that the initial deformed shape of the panel can be determined using the total elasticity of each viscoelastic material ( $E_{total} = E_m + E_s$ ). The end point (asymptotic) condition of the panel can easily be determined by using the standalone elastic portion of the material stiffness,  $E_s$ . Using these material properties to obtain the beginning and ending conditions of the panel, and ensuring that at all times the total forces in each panel object always sum to the total panel force give reassurance that the method is integrated properly.

While this method is integrated and gives good time dependent results for viscoelastic analysis, it is still a good distance from being a commercially viable component of HyperSizer. First of all, there is no integration with the HyperSizer database or the graphical user interface. This component is essential for a commercial code to let the user avoid having to generate ASCII data files by hand in which it is easy to make a typing mistake and there is no data integrity assurance. Second, material properties at this time must be entered for each ply of each analysis object of the panel concept. For the simple case illustrated here, there are only three analysis objects consisting of one ply each. For a more complicated geometry, many more ply definitions would be required. For example, a hat stiffened panel, with an 18 ply facesheet and 6 ply hat stiffener requires over 60 ply

material definitions. That is 60 definitions for a single panel! If there are hundreds of panels across a vehicle surface, the amount of input definition data becomes prohibitive. Therefore, more integration and efficiency work is required to make this implementation commercially viable.

### ***GMC Viscoelasticity Implementation in HyperSizer***

With the basic time-stepping and iteration algorithm in place, a callable GMC routine called **EpsEtaGMC** was formed from the **GMCVisc** program written by Dr. Carlos Orozco. This routine was almost a separate program on its own which read its own input data from a data file, kept track of its own history data and basically returned the viscous strain terms ( $\epsilon^v$ ) given the current strains and previous time step viscous strains. The technique proved to work well, though the separation of the routine data from that of the main HyperSizer program could prove to make full integration into the commercial version more difficult.

To perform the GMC based viscoelastic analysis, the procedure is the same as the previous viscoelastic setup except that an additional data file must be setup for the GMC input data. Again, the viscoelastic material properties for each ply was entered into the \*.VMT file, except that the GMC analysis flag was this time entered as '1' instead of '0'. In addition, the input file for the GMC analysis looked like this:

```

4      0      Number of materials, viscoplst. flag

'glass-v'
      2      Number of different temperatures
      1 0.2100D+02 Temperature (i)
0.3447D+05 0.3447D+05 0.2872D+20      Es,Em,Eta
0.2000D+00 0.2000D+00      Nuaxial,Nutrans
0.1000D-05 0.1000D-05      alphaxial, alphtrans
      2 0.1210D+03 Temperature (i)
0.3447D+05 0.3447D+05 0.2872D+20      Es,Em,Eta
0.2000D+00 0.2000D+00      Nuaxial,Nutrans
0.1000D-05 0.1000D-05      alphaxial, alphtrans

'epoxy-r (cold) '
      2      Number of different temperatures
      1 0.2100D+02 Temperature (i)
0.4057D+04 0.1990D+03 0.1470D+06      Es,Em,Eta
0.3110D+00 0.3110D+00      Nuaxial,Nutrans
0.1000D-05 0.1000D-05      alphaxial, alphtrans
      2 0.1210D+03 Temperature (i)
0.3080D+04 0.3350D+03 0.4000D+06      Es,Em,Eta
0.3170D+00 0.3170D+00      Nuaxial,Nutrans
0.1000D-05 0.1000D-05      alphaxial, alphtrans

'epoxy-r (hot) '
      1      Number of different temperatures
      1 0.2100D+02 Temperature (i)
0.3080D+04 0.3350D+03 0.4000D+06      Es,Em,Eta
0.3170D+00 0.3170D+00      Nuaxial,Nutrans
0.1000D-05 0.1000D-05      alphaxial, alphtrans

'epoxy-r (hot) 2'
      1      Number of different temperatures
      1 0.2100D+02 Temperature (i)
0.3080D+04 0.3350D+03 1.000D+20      Es,Em,Eta
0.3170D+00 0.3170D+00      Nuaxial,Nutrans
0.1000D-05 0.1000D-05      alphaxial, alphtrans

```

Initialization Data and  
material definitions

```

3      Number of PLY

0.5000D-02  0.1000D-05      strainamp, strrate
           2          100      100      loadopt, nint, nintact
                   2          50      nstep, nsteps

0.2100D+02  0.1000D+01      reftemp, deltatem
           4      4      ncellsy, ncellsx
3 3 3 3
3 3 3 3
3 3 3 3
3 3 3 3

10      Geometry ID

0.0020
0.0020
0.0020
0.0020
0.5000D+01  0.5000D+01  0.5000D+01  0.5000D+01

4      4      ncellsy, ncellsx
2 2 2 2
2 2 2 2
2 2 2 2
2 2 2 2

10      Geometry ID

0.0020
0.0020
0.0020
0.0020
0.5000D+01  0.5000D+01  0.5000D+01  0.5000D+01

4      4      ncellsy, ncellsx
3 3 3 3
3 3 3 3
3 3 3 3
3 3 3 3

10      Geometry ID

0.0125
0.0125
0.0125
0.0125
0.5000D+01  0.5000D+01  0.5000D+01  0.5000D+01

```

Subcell Definitions

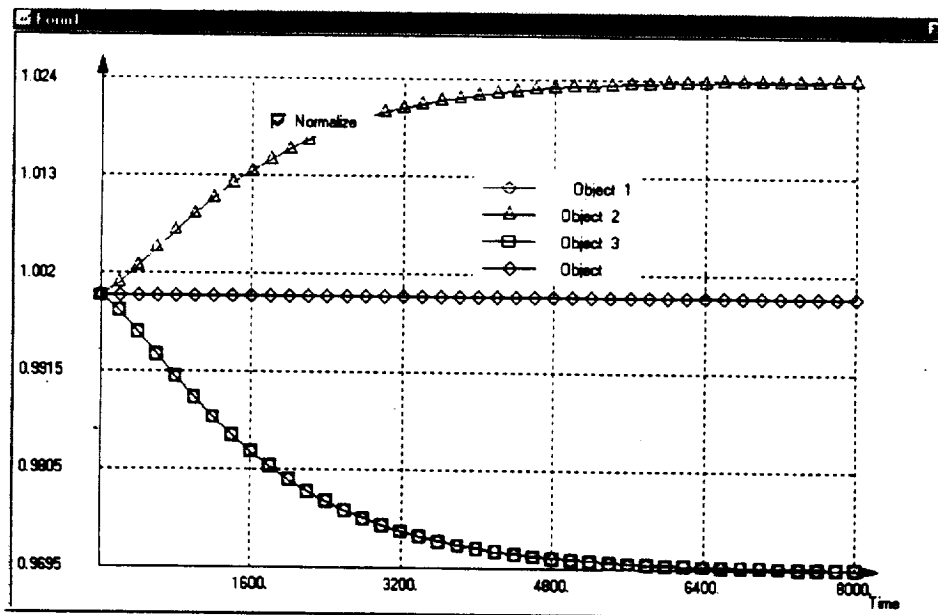
This file, located in the project Material directory is called <Project>.GMC. Note that the number of plies defined in this GMC file must match the number of plies defined in the \*.VMT file. For more information about the GMC file, see the GMCVisc documentation listed in Appendix B.

Note that there is currently a disconnect in the material stiffness calculation between the \*.VMT file and the \*.GMC file. This disconnect is in the afact that the material stiffnesses are not passed back from the GMC module and therefore the only stiffnesses that HyperSizer knows about are those defined in the VMT file. For homogeneous

viscoelastic materials, this is not a problem. However, for composite plies with two or more dissimilar materials, there is no way for HyperSizer to know about the stiffnesses that the GMC module is using. If this research effort is continued, this deficiency must be addressed.

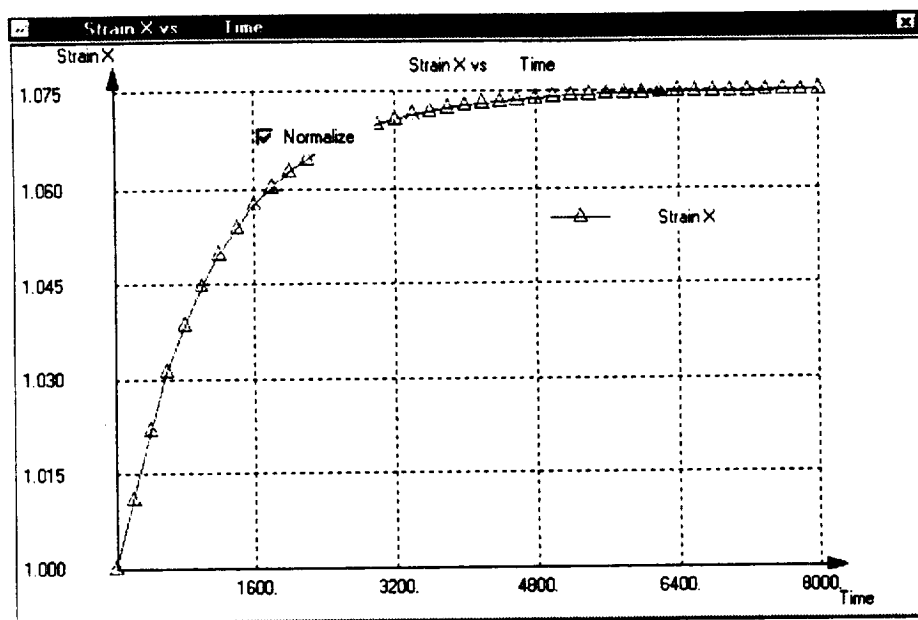
## Results

In order to test the implementation of the GMC viscoelasticity, a problem identical to the three-ply model from the previous section was setup with HyperSizer using the GMC methodology. The same \*.VMT file from the previous problem was used (with the GMC analysis flag set to '1') and the \*.GMC file as shown on the previous page was used. The run parameters and material properties were identical. The stress plots from the GMC based HyperSizer analysis are shown below.



If this plot is compared to the non-GMC HyperSizer results or the Multiplies results shown in the previous section, the results are nearly identical.

The strain results, which also match up very well with previous results are shown on the following page.



Again, this sample problem indicates that the HyperSizer implementation seems to work and appears to be feasible for even more large scale and complex panel concepts. The same arguments hold for this case as far as checking the solution for accuracy by checking the upper and lower bounds on the solution using the total stiffness and elastic stiffness respectively.

While this procedure has shown that the GMC implementation is feasible for a full panel, it also points out that there is still a considerable amount of work to be done to make it into a commercially viable version. First, there is the disconnect between the stiffnesses used by the GMC and those specified in the \*.VMT file. (For this case, the difficulty was overcome by making each ply homogeneous and making sure that the properties matched between the GMC file and the VMT file.) Second, the amount of data that must be generated for each ply in the GMC file is prohibitive and must be generated by hand. For this simple case of a single panel with only three plies, this is no problem, but for a problem with hundreds of panels, each with dozens of plies, the amount of input data would become unmanageable.

## ***New Technology***

Several new and innovative techniques were developed and improved during the course of this research.

First, the GMC method, originally developed by Dr. Jacob Aboudi, was extended by Dr. Carlos Orozco of UVA to include viscoelastic material analysis. In addition to just adding the capability, Dr. Orozco was able to substantially speed up the process and reduce the very large initial memory requirements. This was in an effort to make the code feasible for inclusion into HyperSizer. This capability should prove to be very valuable in performing very accurate, detailed analysis for fiber matrix composites. While we believe that there is still work to be done in order to make this capability commercially, there were substantial advances and improvements made in this technology.

Second, a substantial amount of effort went into the addition of the time-stepping algorithm into HyperSizer. While time stepping and iterative schemes are nothing new, their inclusion into HyperSizer is. This advance in HyperSizer went a long way into showing that this kind of analysis is completely compatible with HyperSizer's unique methods for analyzing and optimizing complex built-up stiffened panel and beam structures. In addition to adding the time-stepping to HyperSizer, a way of increasing the accuracy of the analysis without sacrificing accuracy was devised in the unique sub-iteration technique discussed in Appendix A.

Finally, this research has demonstrated that viscoelasticity and GMC are feasible concepts for inclusion into HyperSizer. Furthermore, the methods developed open the door for more advanced technologies such as viscoplasticity and post-buckling strength analysis. While more work needs to be done to make these things commercially, this research should prove to be a good base from which to work.



# INCORPORATION OF A REVERSIBLE HEREDITARY VISCOELASTIC MODEL WITHIN GMC

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The following is a summary of the main relationships that I have used to incorporate the viscoelastic constitutive model of Ref. [1] within the context of the generalized method of cells (GMC).

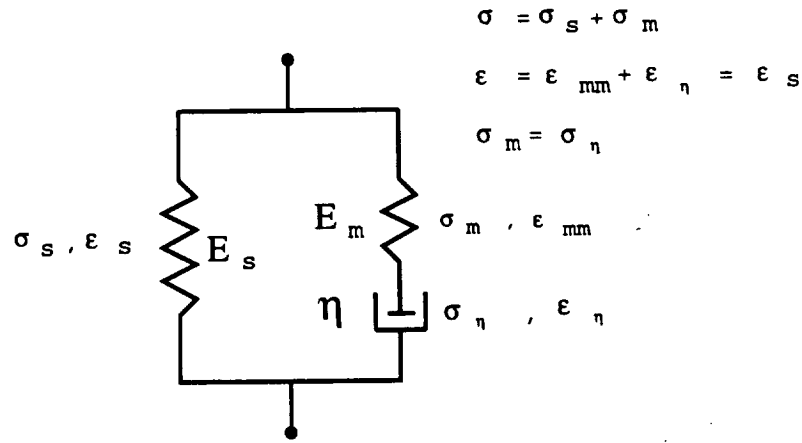


Figure 1: Standard linear three element model.

Equations (4) and (5) of Ref. [1] are reproduced here in matrix form as:

$$\dot{\epsilon}_s = C_s^{-1} \dot{\sigma}_s + \theta_s \dot{T} \quad (1)$$

$$\dot{\epsilon}_m = C_m^{-1} \dot{\sigma}_m + \theta_m \dot{T} + \dot{\epsilon}_\eta \quad (2)$$

where  $C_s$  is the elastic stiffness matrix of a material with modulus of elasticity  $E_s$ ;  $C_m$  is the elastic stiffness matrix of a material with modulus of elasticity  $E_m$ ; and the  $\theta$ s are as follows:

$$\theta_s \equiv \frac{\partial C_s^{-1}}{\partial T} \sigma_s + w \quad (3)$$

$$\theta_m \equiv \frac{\partial C_m^{-1}}{\partial T} \sigma_m + w \quad (4)$$

where

$$w \equiv \left\{ \frac{w_{tan}}{3}, \frac{w_{tan}}{3}, \frac{w_{tan}}{3}, 0, 0, 0 \right\}^T \quad (5)$$

with

$$w_{tan} \equiv w + \frac{\partial w}{\partial T} \Delta T \quad (6)$$

In equations (1) through (6) subindex  $s$  refers to the single spring element in Figure 1 and subindex  $m$  refers to the Maxwell element in Figure 1.  $w$  represents the coefficient of thermal expansion of the material.

Solving for the stress rates in (1) and (2) we get:

$$\dot{\sigma}_s = C_s(\dot{\epsilon}_s - \theta_s \dot{T}) \quad (7)$$

$$\dot{\sigma}_m = C_m(\dot{\epsilon}_m - \dot{\epsilon}_\eta - \theta_m \dot{T}) \quad (8)$$

Now, since

$$\dot{\sigma} = \dot{\sigma}_m + \dot{\sigma}_s \quad (9)$$

Eqns. (7) and (8) can be combined to yield:

$$\dot{\sigma} = C_E \dot{\epsilon} - C_m \dot{\epsilon}_\eta - C_m \theta_m \dot{T} - C_s \theta_s \dot{T} \quad (10)$$

where  $C_E \equiv C_s + C_m$ .

Now, using the equal poisson ratio assumption of Ref. [1], the stiffness matrices can be written as:

$$\begin{aligned} C_m &\equiv E_m N \\ C_s &\equiv E_s N \\ C_E &\equiv E N \end{aligned} \quad (11)$$

where  $E \equiv E_s + E_m$ .

Using (3) and (4), Eqn. (10) can be rewritten as:

$$\dot{\sigma} = C_E \dot{\epsilon} - C_m \dot{\epsilon}_\eta - C_s \hat{\theta} \dot{T} \quad (12)$$

where

$$\hat{\theta} \equiv \theta_s + \frac{E_m}{E_s} \theta_m = \frac{\partial C_s^{-1}}{\partial T} \sigma_s + \frac{E_m}{E_s} \frac{\partial C_m^{-1}}{\partial T} \sigma_m + \frac{E}{E_s} w \quad (13)$$

(Note that this is not the same  $\hat{\theta}$  of Ref. [1].)

Eqn.(12) can also be written:

$$\dot{\sigma} = C_E(\dot{\epsilon} - \dot{\epsilon}_\eta - \dot{\epsilon}^T) \quad (14)$$

where

$$\dot{\epsilon}_\eta \equiv \frac{E_m}{E} \dot{\epsilon}_\eta \quad (15)$$

and

$$\dot{\epsilon}^T \equiv \frac{E_s}{E} \hat{\theta} \dot{T} \quad (16)$$

Eqn. (14) constitutes a constitutive law that is analogous to that of plasticity or viscoplasticity. This form lends itself to easy implementation within the context of GMC without having to change the assembly of the matrices associated with the GMC procedure.

# 1 Formulation Without Internal Variables

From equilibrium in the Maxwell element in Figure 1, we have:

$$\sigma_m = \sigma_\eta \quad (17)$$

or

$$\sigma_m = \eta \dot{\epsilon}_\eta \quad (18)$$

where, in analogy with (11)

$$\eta \equiv \eta N \quad (19)$$

Eqn. (12) can now be written as:

$$\dot{\sigma} = C_E \dot{\epsilon} - C_m \eta^{-1} \sigma_m - C_s \hat{\theta} \dot{T} \quad (20)$$

or, in view of (9)

$$\dot{\sigma} = C_E \dot{\epsilon} - C_m \eta^{-1} (\sigma - \sigma_s) - C_s \hat{\theta} \dot{T} \quad (21)$$

Solving now for  $\dot{\epsilon}$  in (21), we get:

$$\dot{\epsilon} = \frac{E_m}{E} C_m^{-1} \dot{\sigma} + \frac{E_m}{E} \eta^{-1} (\sigma - \sigma_s) + \frac{E_s}{E} \hat{\theta} \dot{T} \quad (22)$$

which can be shown to be equivalent to Eqn. (17) of Ref. [1].

## 2 Solution of the Differential Equation

Making use of the fact that:

$$\sigma_s = C_s \epsilon \quad (23)$$

and using relationships (11) and (19), Eqn. (21) can be written:

$$\dot{\sigma} = C_E \dot{\epsilon} + \frac{E_m}{\eta} C_s \epsilon - \frac{E_m}{\eta} \sigma - C_s \hat{\theta} \dot{T} \quad (24)$$

This is a first order differential equation that can be solved either for  $\sigma$  or for  $\epsilon$ .

A simple forward Euler scheme can be used to solve (24) by writing:

$$\sigma(t + \Delta t) \approx \sigma(t) + \left\{ C_E \dot{\epsilon}(t) + \frac{E_m}{\eta} C_s \epsilon(t) - \frac{E_m}{\eta} \sigma(t) - C_s \hat{\theta}(t) \dot{T} \right\} \Delta t \quad (25)$$

or, in view of (16),

$$\sigma(t + \Delta t) \approx \sigma(t) + \left\{ C_E [\dot{\epsilon}(t) - \dot{\epsilon}^T(t)] + \frac{E_m}{\eta} C_s \epsilon(t) - \frac{E_m}{\eta} \sigma(t) \right\} \Delta t \quad (26)$$

where  $\dot{\epsilon}^T$  is the thermal strain rate.

A slightly more accurate way of solving Eqn. (24) numerically consists of finding its analytical solution assuming that either the temperature and strain history, or the

temperature and stress history is known. In what follows this is done for the former case (i.e., the temperature and strain histories are known).

To this end rewrite Eqn. (24) as:

$$\dot{\sigma} + \gamma\sigma = \alpha\dot{\epsilon} + \beta\epsilon - \delta(t)\dot{T} \quad (27)$$

where

$$\gamma \equiv \frac{E_m}{\eta}$$

$$\alpha \equiv C_E$$

$$\beta \equiv \frac{E_m}{\eta} C_s$$

$$\delta(t) \equiv C_s \hat{\theta}(t)$$

This is an O.D.E. of the form:

$$y' + f(x)y = r(x)$$

whose exact solution is:

$$y(x) = e^{-h} \left[ \int e^h r(x) dx + C \right]$$

where  $h \equiv \int f(x) dx$ .

Therefore the solution of (26) is:

$$\sigma(t) = e^{-\gamma t} \left[ \int_0^t \alpha e^{\gamma \tau} \dot{\epsilon} d\tau + \int_0^t \beta e^{\gamma \tau} \epsilon d\tau - \int_0^t \delta(\tau) e^{\gamma \tau} \dot{T} d\tau \right] \quad (28)$$

where

$$\delta(t) \equiv C_s \left\{ \frac{\partial C_s^{-1}}{\partial T} \sigma_s + \frac{E_m}{E_s} \frac{\partial C_m^{-1}}{\partial T} \sigma_m + \frac{E}{E_s} w \right\} \quad (29)$$

After some algebraic manipulations, an incremental version of Eqn. (28) is found as:

$$\sigma(t + \Delta t) = e^{-\gamma \Delta t} \sigma(t) + e^{-\gamma \frac{\Delta t}{2}} \left\{ \alpha \dot{\epsilon} \left( t + \frac{\Delta t}{2} \right) + \beta \epsilon \left( t + \frac{\Delta t}{2} \right) - \delta \left( t + \frac{\Delta t}{2} \right) \dot{T} \left( t + \frac{\Delta t}{2} \right) \right\} \Delta t \quad (30)$$

This expression must be used instead of Eqn. (28) when the strain and temperature histories are not available in analytical form.

In terms of the original parameters of the problem, Eqn. (30) can be rewritten as:

$$\sigma(t + \Delta t) = e^{-\frac{E_m}{\eta} \Delta t} \sigma(t) + e^{-\frac{E_m}{\eta} \frac{\Delta t}{2}} \left[ C_E (\dot{\epsilon} - \dot{\epsilon}^T) + \frac{E_m}{\eta} C_s \epsilon \right]_{t+\Delta t/2} \Delta t \quad (31)$$

Note the similarity between Eqn. (31) and Eqn. (40) in Ref. [1]. This expression has been programmed within the new version of GMC for both the isothermal and the nonisothermal case.

The figure that follows illustrates the results obtained with the new viscoelastic version of GMC. It corresponds to a relaxation test for TIMETAL 21S at 565 degrees C. It can be seen that the GMC approximation is very close to the exact solution.

The nonisothermal case requires an additional approximation since the quantity  $\delta$  in (30) is not known at time  $t + \frac{\Delta t}{2}$ . This quantity is then approximated by  $\delta(t)$ .

### 3 Nonisothermal Case

The nonisothermal case requires additional work on Eqn. (29) to make it suitable for the Euler solution procedure.

Using (16) and (13)  $\dot{\epsilon}^T$  in (31) can be written as:

$$\dot{\epsilon}^T = \left( \frac{E_s}{E} \theta_s + \frac{E_m}{E} \theta_m \right) \dot{T} \quad (32)$$

where  $\theta_s$  and  $\theta_m$  are given by (3) and (4).

Now, for a homogeneous isotropic material,  $C_s^{-1}$  corresponds to a compliance matrix given by:

$$C_s^{-1} = E_s^{-1} L \quad (33)$$

where

$$L \equiv \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \quad (34)$$

The partial derivative in Eqn. (3) can then be expressed as:

$$\frac{\partial C_s^{-1}}{\partial T} = -E_s^{-2} \frac{\partial E_s}{\partial T} L + E_s^{-1} \frac{\partial L}{\partial T} \quad (35)$$

where

$$\frac{\partial L}{\partial T} = \frac{\partial \nu}{\partial T} L_0 \quad (36)$$

and

$$L_0 \equiv \begin{bmatrix} 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \quad (37)$$

The parameter  $\theta_s$  can now be expressed as:

$$\theta_s = \left( -E_s^{-2} \frac{\partial E_s}{\partial T} L + E_s^{-1} \frac{\partial \nu}{\partial T} L_0 \right) \sigma_s + w \quad (38)$$

The analogous expression for  $\theta_m$  is then:

$$\theta_m = \left( -E_m^{-2} \frac{\partial E_m}{\partial T} L + E_m^{-1} \frac{\partial \nu}{\partial T} L_0 \right) \sigma_m + w \quad (39)$$

It is necessary now to find expressions for  $\sigma_s$  and  $\sigma_m$  to complete the derivation. Given the stress rate  $\dot{\sigma}$  and the thermal strain rate  $\dot{\epsilon}^T$  at a given time  $t$ , the new modified viscous strain can be obtained from (14) as:

$$\dot{\epsilon}_\eta = -C_E^{-1} \dot{\sigma} + \dot{\epsilon} - \dot{\epsilon}^T \quad (40)$$

Once the new total stress and viscous strain rate are known,  $\sigma_s$  and  $\sigma_m$  can then be obtained as:

$$\sigma_m = \frac{E}{E_m} C_\eta \dot{\epsilon}_\eta \quad (41)$$

and

$$\sigma_s = \sigma - \sigma_m \quad (42)$$

The preceding derivation provides a complete account of all the expressions needed to implement an Euler-based numerical integration procedure for the multiaxial nonisothermal viscoelastic model as presented in [1].

## References

- [1] A. F. Saleeb and S. M. Arnold. A general reversible hereditary constitutive model: Part i theoretical developments. Technical Report Preprint, NASA, 1997.

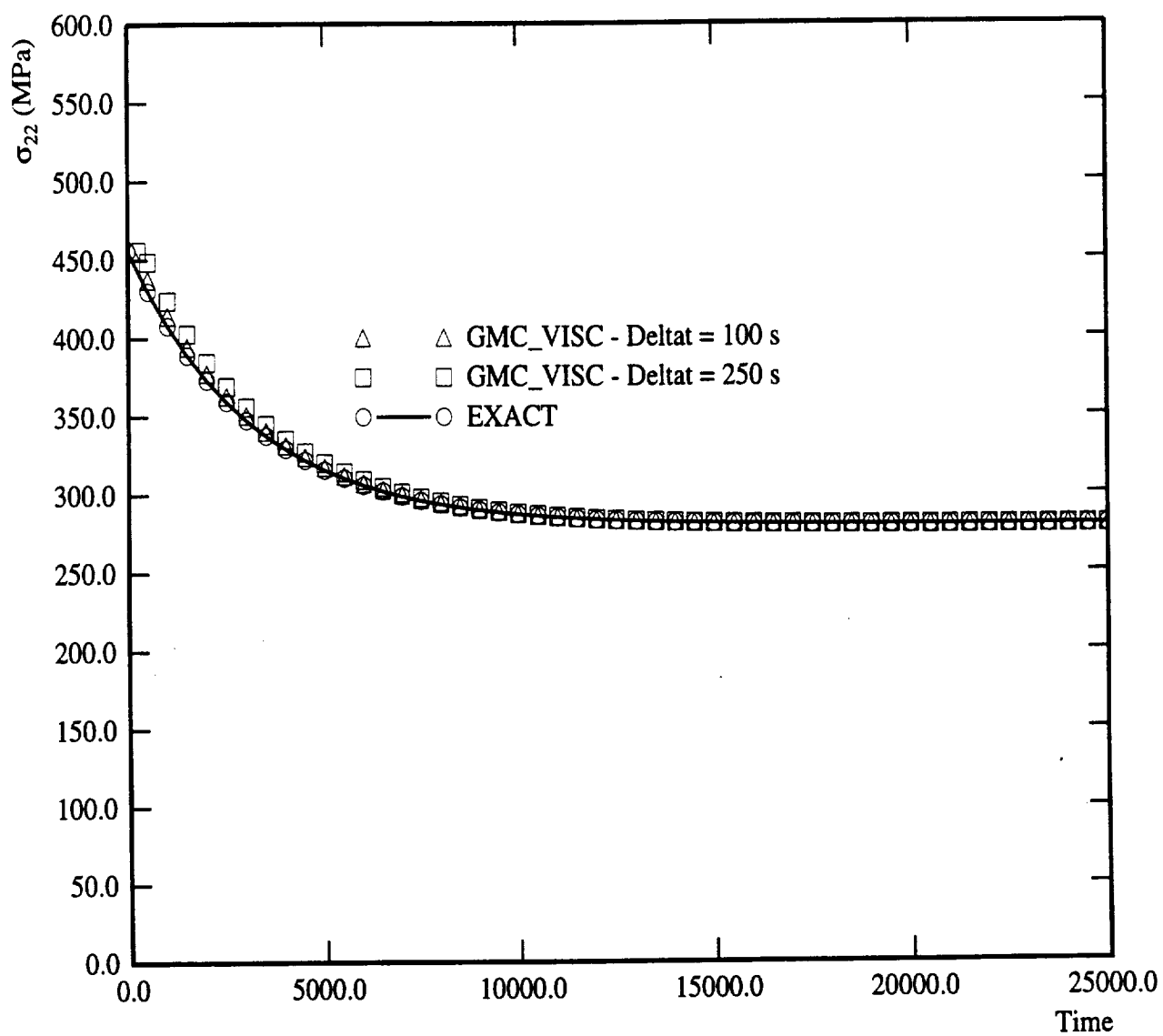


Figure 2: Relaxation results for TIMETAL 21S at 565 degrees C.





## **PROGRESS ON LAMINATE/PANEL VISCOELASTIC INTEGRATION**

NASA NRA Contract NAS3-97051

Collier Research Corporation

January 14, 1998

### **Abstract**

Task number 2 of the NRA contract involves incorporating fiber/matrix composite viscoelastic behavior into the commercial sizing/analysis code called HyperSizer. Dr. Carlos Orozco has advanced this research by applying the General Method of Cells (GMC), originally developed by Dr. Jacob Aboudi, to the viscoelastic behavior of general composite materials. In the current effort, an algorithm to incorporate this GMC methodology with either a single ply, a multi-ply laminate or a full stiffened panel has been developed and is currently under development and testing. Because the analysis of a full structural panel is a complex problem with a limited number of available exact solutions, a simplified formulation was developed to predict and analyze the viscoelastic behavior of 1-D strip elements composed of multiple "plies." By analyzing a simplified problem for which closed form, analytical solutions were known, the numerical algorithm was tested and its performance was characterized, and is presented herein. The numerical algorithm for a full panel subjected to specified constant stress ("creep"), specified constant strain ("stress relaxation"), specified transient stress or specified transient strain loading is presented. A number of closed form solutions are presented against which the simplified numerical formulation was tested in the form of a stand-alone, prototype computer code. The comparisons indicate that the methodology is consistent and 1<sup>st</sup> order accurate, that is, the solution accuracy is directly proportional to the magnitude of the time step. Also, while the forward Euler time stepping scheme is conditionally stable, the time step restriction is easily characterized as a function of the viscoelastic material properties and can be easily automated. In addition, a sub-iteration scheme was developed to allow multiple time steps to be performed for the individual plies before advancing the global time step. This minimizes the number of switches between different materials and plies. This modification to the formulation is shown to improve the efficiency of the analysis without compromising the accuracy. The next steps are to include the GMC subroutine and full panel formulation into the prototype code. This will allow characterization of GMC behavior with the numerical formulation presented here. The final step is to integrate GMC and the viscoelastic methodology into HyperSizer itself.

## **1. Introduction**

One of the primary tasks of the NRA contract was to incorporate the effects of viscoelasticity into the commercial analysis / optimization code called HyperSizer. In particular, we wish to model the viscoelastic behavior of orthotropic materials such as polymeric or metal matrix composites. In order to accomplish this, we are using the General Method of Cells (GMC) originally developed by Dr. Jacob Aboudi to model the micromechanical behavior of the fiber/matrix composites. This research is aimed at incorporating the micromechanical response obtained using GMC into a macromechanical model predicting the behavior of a single ply, a laminate consisting of multiple plies, a panel consisting of multiple laminates (such as a corrugated panel) and ultimately, a structure composed of multiple panels.

The first step in this integration lies in determining the best way to use the GMC method to extend from a single ply to a laminate composed of multiple plies. In order to accomplish this, it is useful to take a step back from the full plate theory analysis to a simpler 1-D analysis involving a simple bar exhibiting “strip” behavior as opposed to “plate” behavior. This bar could be composed of one or many plies, exposed to either tension or compression. This simple analysis allows us to derive analytical or “exact” solutions to which we can compare our numerical methods for analyzing laminates and thus determining if the method that we are using is correct.

This report details the development of an algorithm for integrating the micromechanical, single material analysis methodology into a laminate response. A series of analytical solutions were derived and compared to the simplified 1-D numerical model of this algorithm in a computer code and the results are presented and discussed below. The final section contains a discussion of future work and the next direction for this research effort.

Before going into detail about the work performed here, we present a short background leading to the development of the proposed algorithms.

## **2. Background**

Viscoelasticity is a complex time-dependent phenomena that becomes very difficult to model in closed form for any type of composite material. Dr. Carlos Orozco has extended the GMC methodology of Dr. Aboudi to perform viscoelastic analyses using a simple forward Euler scheme to step the viscoelastic solution in time. His implementation is based on an observation that the stress rate,  $\dot{\sigma}$  in a viscoelastic member can be expressed as:

$$\dot{\sigma} = E(\dot{\epsilon} + \hat{\dot{\epsilon}}_{\eta} + \dot{\epsilon}_T)$$

$$\hat{\dot{\epsilon}}_{\eta} = \frac{E_m}{E} \dot{\epsilon}_{\eta}$$

where  $E$  is the overall stiffness,  $E_m$  is the viscoelastic stiffness,  $\dot{\epsilon}$  is the overall strain rate,  $\dot{\epsilon}_{\eta}$  is the strain rate in the viscous damper, and  $\dot{\epsilon}_T$  is the thermal strain rate. The period superscript notation (i.e.  $\dot{\epsilon}$ ) indicates derivative with respect to time ( $d\epsilon/dt$ ). When cast in this form, it is apparent that the viscous strain quantity,  $\hat{\dot{\epsilon}}_{\eta}$  acts in exactly the same manner as the thermal strain, therefore, Carlos' idea is to implement the viscous strain for a laminate in the same well established way that HyperSizer implements thermal strain.

Carlos' implementation of GMC takes the form of a FORTRAN subroutine which, at each time step, returns the quantity  $\hat{\epsilon}_{\eta,n+1} = f(\Delta t, \epsilon_n, \epsilon_{n+1})$  where the subscript  $n$  denotes the time step number and  $\Delta t$  is the magnitude of the current time step. The quantity  $\hat{\epsilon}_{\eta}$ , calculated for each ply, is then assembled into a laminate viscoelastic response using an equation similar to that given in equation 10 from Ref. [1] This equation is used to assemble ply level thermal strains into a laminate level thermal response. The proposed algorithm is listed in detail in the Formulation section below.

### 3. Purpose

The purpose of the current effort is to:

- Develop an algorithm for integrating the GMC micromechanical viscoelastic model into the HyperSizer commercial sizing/analysis software.
- Demonstrate the effectiveness of this algorithm by developing a simplified 1-D analog and applying the simplified model to problems with known solutions involving 1 or 2 viscoelastic or elastic plies.
- Develop a testbed that can be used to test modifications to the proposed algorithm which may improve the efficiency. For example, one such modification is the addition of an arbitrary number of sub-steps to be performed at the ply level before updating the laminate. With the current implementation of the GMC subroutine, this has the advantage of limiting the number of times the GMC code must switch from one ply material to another and thus limiting the number of matrix inversions. The simplified prototype model gives a way of testing such proposed modifications.
- Provide a framework for implementing an isotropic viscoelastic material in HyperSizer independent of the GMC procedure. This will involve extending the simplified 1-D method to include aspects of plate and/or beam theory such as bending and Poisson effects.

## 4. Formulation

There are two algorithm formulations presented. For each formulation, there are two sub-cases. The first algorithm is for an implementation of a full panel formulation as in HyperSizer. The second formulation involves the simplified 1-D analog from which the exact results are taken. For each formulation, the sub-cases involve either specified stress fields or specified strain fields.

In addition to the algorithm formulation, there were a number of analytical or “exact” solutions identified for comparison to the numerical algorithms. These exact solutions are identified and listed in appendix A.

### 4.1 *HyperSizer (full panel) Formulation*

The full panel formulation is divided into the sub-cases of either response to a specified stress field or specified strain field.

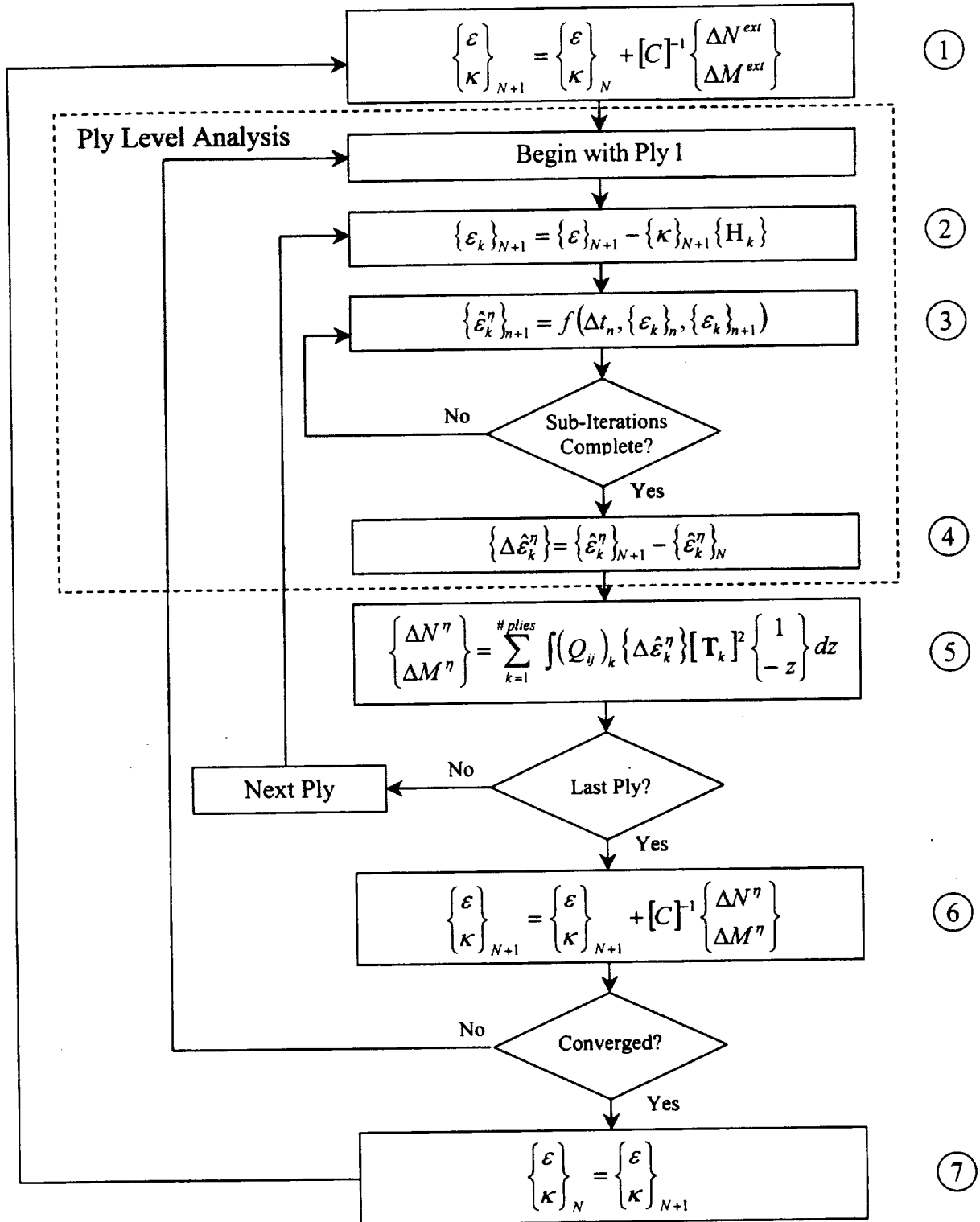
#### 4.1.1 Specified Stress Formulation

In this scenario, the stress field (or panel forces) is specified as a function of time and the algorithm determines the resulting strain field. A special case of this type of analysis called “creep,” occurs when the specified stress field is constant. The algorithm is laid out in the attached Flowchart 1.

In this (and all following flowcharts), the subscript capital N represents the global time step, lower case n represents the time step for each sub-iteration. All quantities with lower case k subscripts represent single ply level quantities and quantities without this subscript are laminate level quantities. (i.e.  $\{\epsilon_k\}$  is a single ply strain field while  $\{\epsilon\}$  represents a laminate strain field)

The following notes apply to the step numbers shown in Flowchart 1:

1. The first step is to calculate the elastic response of the **laminate** given the change in external forces from the previous to the current time step. [C] is the laminate stiffness matrix.
2. Using the strain field at the current time step, calculate the individual ply strains using the laminate strain and curvature field.



Flowchart 1: HyperSizer (full panel) specified stress viscoelastic formulation

3. Using either GMC or some form of standalone subroutine, calculate the viscous strain quantity  $\hat{\epsilon}_\eta$ . This step is performed at each sub-iteration and the time step passed to this function is the time step associated with the sub-iteration (i.e.  $\Delta t_n = \Delta t / (\# \text{ of sub-iterations})$ ). The rationale for using these sub-iterations is to allow multiple time steps for each ply while limiting the number of times a new ply must be loaded. For a homogeneous viscoelastic material, the GMC routine is not required to advance the time step, and the viscous strain quantity is calculated from:

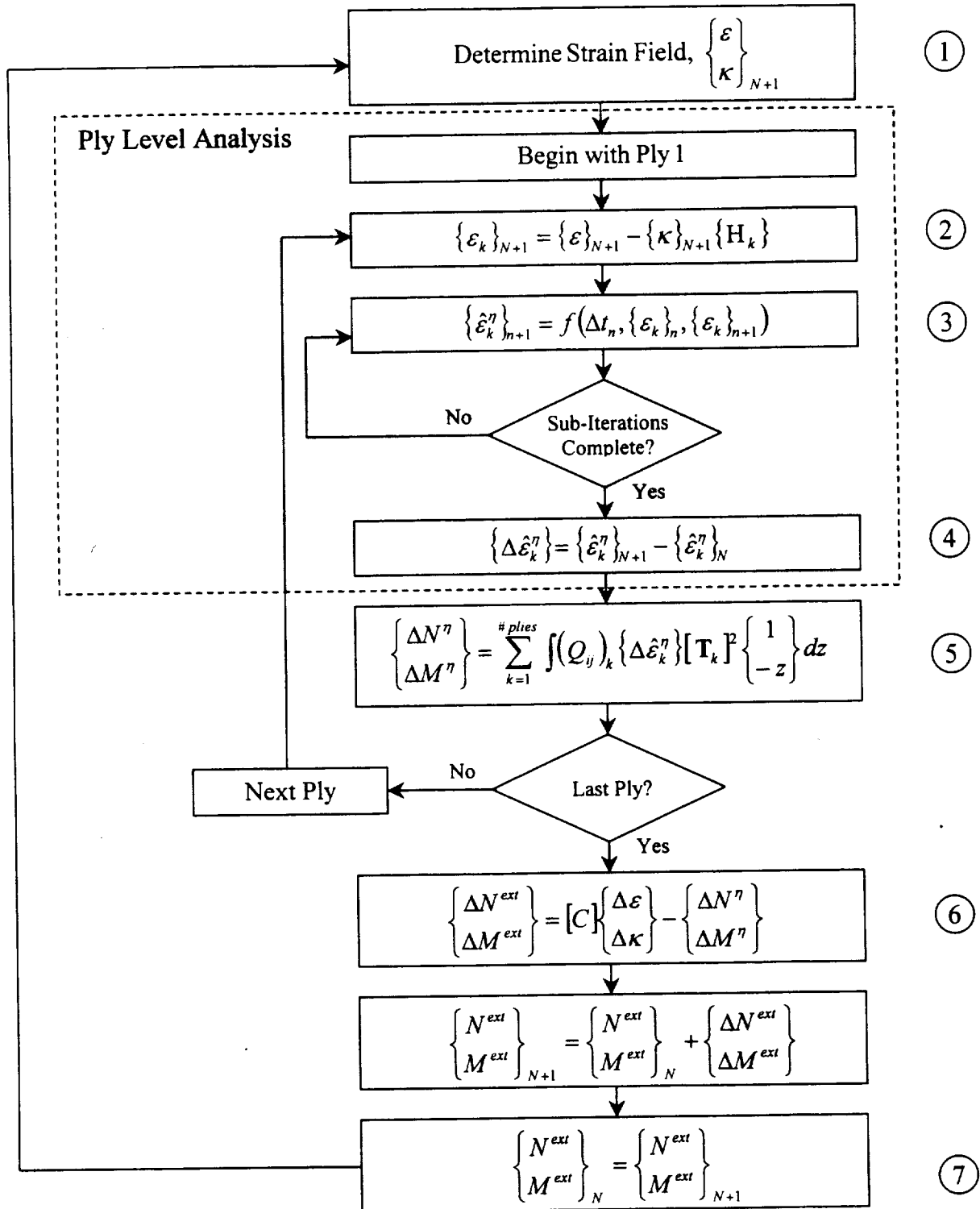
$$\{\epsilon_k^\eta\}_{n+1} = \{\epsilon_k^\eta\}_n + \frac{E_m \Delta t}{E \eta} [\mathbf{Q}]_{3 \times 3} [\{\epsilon_k\}_n - \{\epsilon_k^\eta\}_n]$$

4. Calculate the change in the viscous force from the previous global time step to the current global time step.
5. This equation is similar to Equation 10 from Reference [1]. This equation allows us to “assemble” the viscous strain contributions from each ply into a laminate viscous force. This is exactly how ply level thermal strains are currently “assembled” into laminate thermal forces and moments.
6. Update the laminate strains to reflect the change in the viscous forces. Note that the viscous strain calculated in step 3 is dependent on both the current and the previous levels of strain. Therefore, it may be desirable to feed these new updated strains back to step 2 and iterate 1 or more times. It is unclear at this time whether this iteration will be necessary.
7. Advance to the next time step and return to step 1.

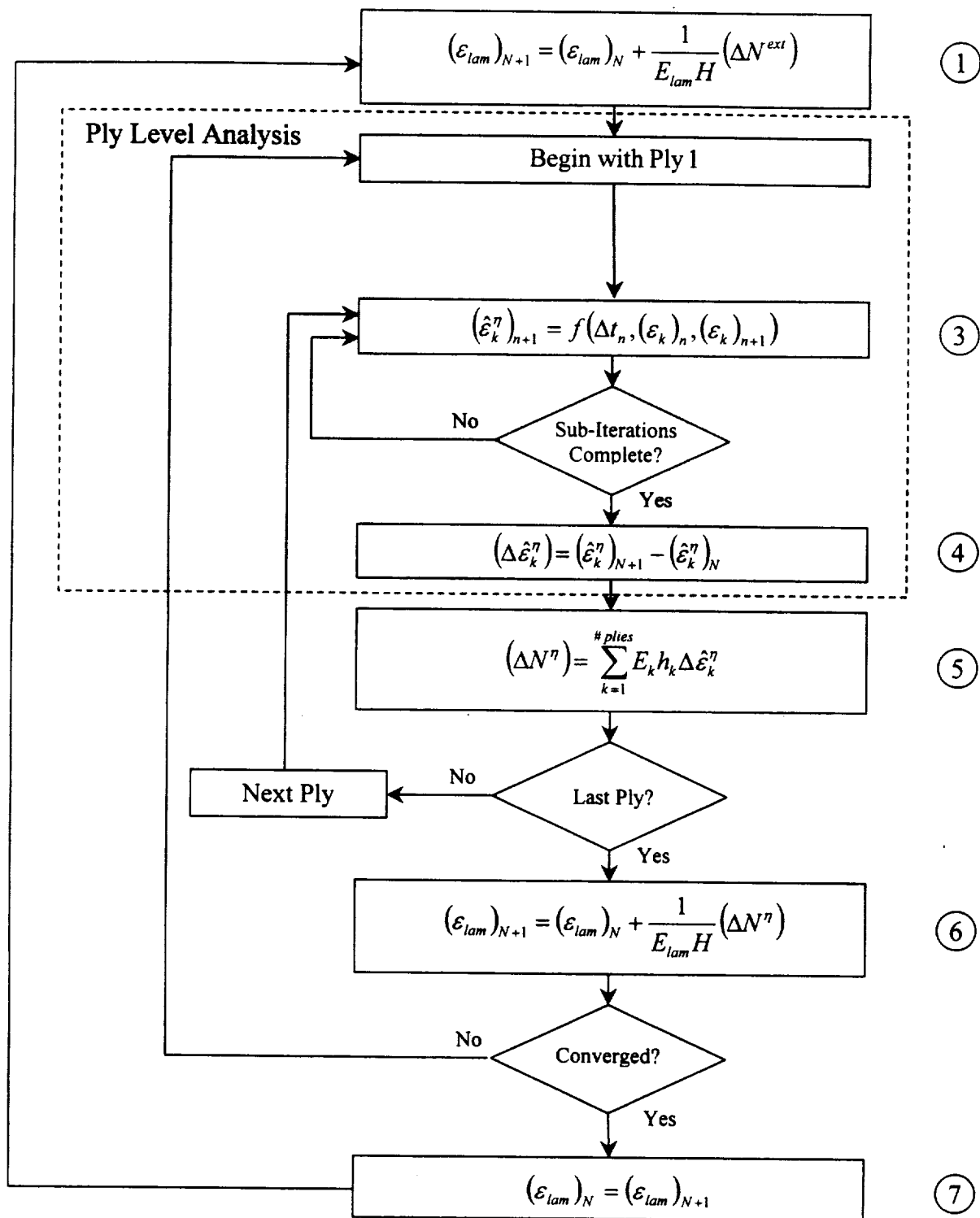
#### 4.1.2 Specified Strain Formulation

In this scenario, the strain field is specified as a function of time and the algorithm determines the resulting stress (or force) field. A special case of this type of analysis occurs when the specified strain field is constant. This situation is called “stress relaxation”. The algorithm is laid out in Flowchart 2.

The algorithm is very similar to the algorithm for specified stress with the exception that instead of updating the strain field at each time step, this algorithm aims to update each plies stress and resulting laminate internal forces and moments. Note that the loop performed at the ply level (steps 3 through 5) which calculates the viscous forces is exactly the same as that of the specified stress formulation.

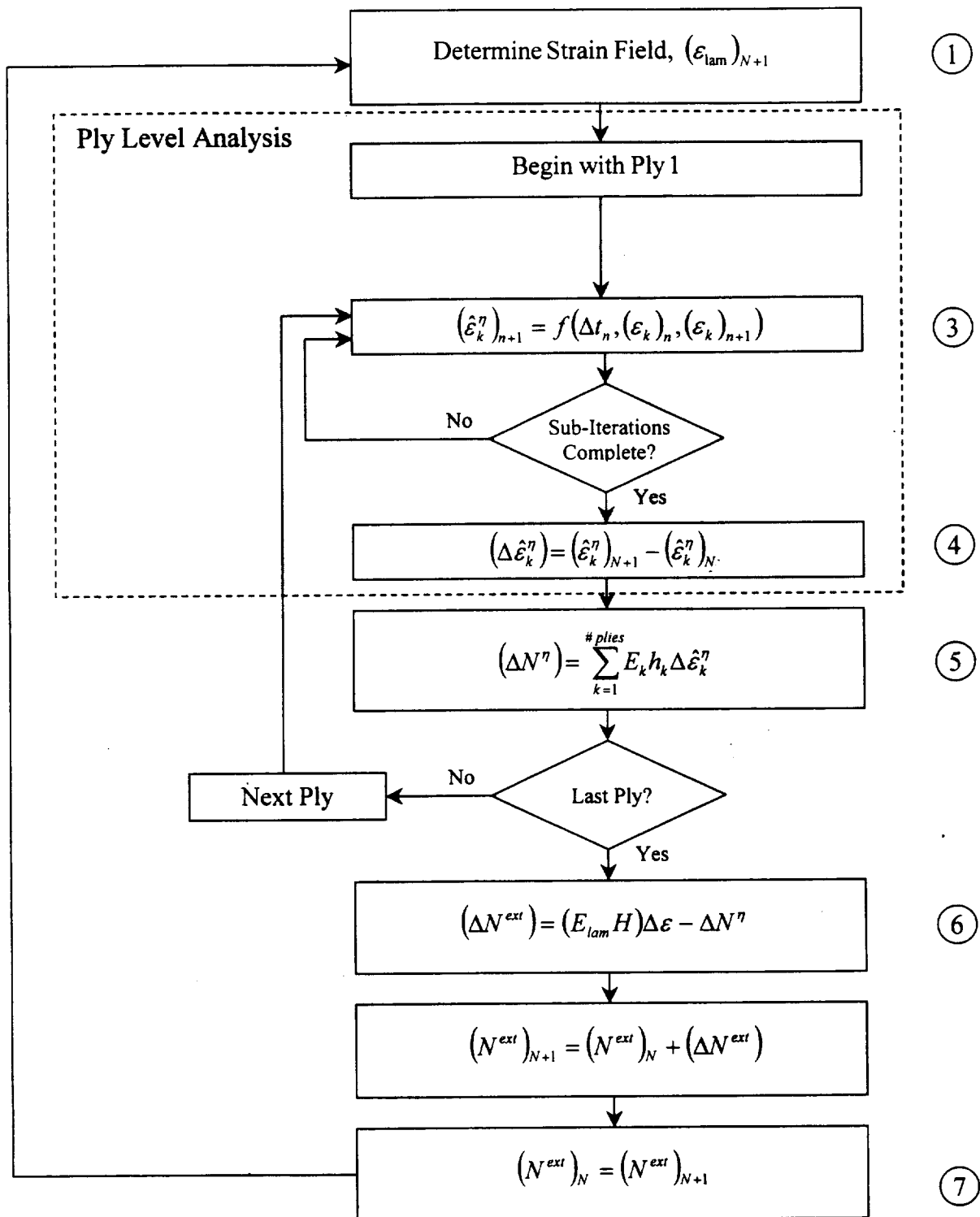


Flowchart 2: HyperSizer (full panel) specified strain viscoelastic formulation



Flowchart 3: Simplified model specified stress (creep) viscoelastic formulation





Flowchart 4: Simplified model specified strain (stress relaxation) viscoelastic formulation

## 4.2 Simplified Formulation

The simplified formulation attempts to model a problem involving 1-D deformation of a laminate composed of multiple plies in the same way as the full panel formulation. In this case, 3-D effects including curvature and Poisson's effect are neglected and the material is characterized by a single physical stiffness,  $E$ . If the ply is viscoelastic, the stiffness,  $E$ , which we can think of as the "instantaneous" stiffness is the sum of the elastic stiffness,  $E_s$  plus the viscoelastic stiffness,  $E_m$ . Also note that for this analysis, there is only one strain as each ply strains at the same rate as the laminate, therefore it makes no sense in the simplified analysis to speak of laminate vs. ply strains. Note that in the simplified analysis, the laminate "stiffness matrix" reduces to the physical stiffness quantity,  $E$ , multiplied by the laminate thickness,  $H$ . In addition, the integral in step 5 (Eq. 10 from Ref. [1]) reduces to:

$$(\Delta N^\eta) = \sum_{k=1}^{\# \text{ plies}} E_k h_k \Delta \hat{\epsilon}_k^\eta$$

### 4.2.1 Specified Stress Formulation

A special case for this formulation occurs when the laminate stress is constant. This special case is called "creep." The formulation, laid out in flowchart number 3, is very similar to the specified stress formulation given for the full panel above. One special note about the simplified algorithm. By using the forward Euler method, the procedure in step 3 to calculate the viscous strain,  $\hat{\epsilon}_\eta$ , is actually independent of the strain at the current time step. In other words, the outer loop (after step 6) is unnecessary for the simplified algorithm, however the step is left in the flowchart to maintain similarity with the full panel formulation.

### 4.2.2 Specified Strain Formulation

This formulation, laid out in flowchart number 4, is exactly analogous to the specified strain formulation for the full panel above.

## 4.3 Analytical Results for Simplified Analysis

The following matrix illustrates the cases for which analytical or "exact" results were derived for comparison to the numerical results from the previous section. There are nine basic cases involving either one or two plies.

Table 1: Loading cases for comparison of analytical and numerical results

	<b>Creep Analysis</b>	<b>Stress Relaxation Analysis</b>	<b>Transient Strain Profile</b>
<b>1 Viscoelastic Ply</b>	Analytical	Analytical	Analytical
<b>1 Viscoelastic &amp; 1 Elastic Ply</b>	Analytical	Analytical	Analytical
<b>2 Viscoelastic plies</b>	Runga-Kutta 4th Order	Analytical	Analytical

The analytical solutions are listed in detail in Appendix A. Note that for the case of creep analysis in 2 viscoelastic plies, instead of solving the resulting system of 2 differential equations in closed form, we chose to model this response using a 4th order Runga-Kutta analysis which, while not exact, is much more accurate than the 1st order Euler numerical integration used in the present effort.

## 5. Results / Discussion

The simplified formulation was implemented in a Windows computer program that simulates any number of plies and any combination of viscoelastic or elastic plies with arbitrary thicknesses. The ply materials used in the analysis are listed in Figure 1. The first material is a generic fiberglass, modeled here as an elastic material with a stiffness of 68,940 MPa. The program is written to be general but no provision is made to model a purely elastic material. Therefore, in order to model an “elastic” material, the viscoelastic damping parameter was set to a very large value ( $1.0 \times 10^{20}$ ). While this is technically a viscoelastic material, the analysis would have to be run for approximately  $10^{15}$  seconds in order to notice any viscoelastic response. Therefore, for all practical purposes, the material is elastic. The second and third plies are both composed of a viscoelastic epoxy taken from Dr. Aboudi’s book. The difference being in the material evaluation temperatures of 22°C (cold) and 140°C (hot).

Figures 2-11 are result plots using combinations of these three plies under the loading conditions laid out in Table 1. In each of the plots, the circle markers represent the numerical results while the solid lines represent the “exact” or analytical result for comparison. The “maximum error” shown for each result is calculated from:

$$e_{\max} = \text{MAX} \left| \frac{P_{\text{numerical}} - P_{\text{exact}}}{P_{\text{exact}}} \right|$$

The obtained results demonstrate that the method is consistent, that is, it converges on the exact solution as the time step is reduced to zero. Figures 2a-e illustrate

this trend as well as the effectiveness of the sub-iteration technique. Because the chosen method of time integration (forward Euler) is first order accurate, one would expect the error to be linearly proportional to the size of the time step. In figures 2a and 2b, the ratio of the time steps is 5.0, and the ratio of the maximum errors is about 5.8. Reducing the time step by another factor of 5.0 in figure 2e reduces the maximum error by a factor of 5.2. This behavior continues as the time step size is reduced to zero, as illustrated in Figure 2g.

a) Elastic ply

<div>Epoxy (cold)</div> <div>Epoxy (Hot)</div>	New Ply	34470.	Elastic Stiffness
	Delete Ply	34470.	Visco Stiffness
		1.000E+20	Damping Coefficient
		0.01000	Thickness
		2.901E+15	Time Constant
	Close	68940.	Stiffness

b) Cold viscoelastic ply

<div>Glass-V</div> <div>Epoxy (Hot)</div>	New Ply	4057.	Elastic Stiffness
	Delete Ply	199.0	Visco Stiffness
		147000.	Damping Coefficient
		0.05000	Thickness
		738.7	Time Constant
	Close	4256.	Stiffness

c) Hot viscoelastic ply

<div>Glass-V</div> <div>Epoxy (cold)</div>	New Ply	3080.	Elastic Stiffness
	Delete Ply	335.0	Visco Stiffness
		400000.	Damping Coefficient
		0.05000	Thickness
		1194.	Time Constant
	Close	3415.	Stiffness

Figure 1: Ply properties using in viscoelastic analysis tests

In addition to the convergence of the algorithm with decreasing time step, the sub-iterations are also effective in reducing the error. As shown in Figures 2a and 2b, reducing the time step by 5 decreases the error by a factor of 5, but Figure 2c demonstrates that increasing the number of sub-steps by a factor of 5 has almost the same effect. In fact the reduction of error obtained in this case is more than 10 times, meaning that this is a better solution than that of Figure 2b. While the results obtained by increasing the number of sub-iterations are not always better, it was observed that they were always comparable. It appears that there is some "optimum" value for the number of sub-iterations which we did not determine, but between 5 and 10 generally gave very good results. One word of caution. One of the drawbacks of using Euler integration is that the method is conditionally stable. If the time step is too large, oscillations in the solution occur as illustrated in Figure 2f. The practical limit on the time step size is the minimum time constant of the viscoelastic materials in the laminate. This time constant,  $\tau$ , is calculated from the viscoelastic properties as:

$$\tau = \frac{\eta}{E_m}$$

where  $\eta$  is the viscoelastic damping coefficient and  $E_m$  is the viscoelastic stiffness. For the material shown in Figure 2f, the time constant is 738 seconds and the time step used is 1000 seconds.

Although convergence results are not shown for the rest of the cases, all of the loadings and ply combinations exhibit the same convergence and stability behavior as that shown in figure 2.

Figures 2-11 demonstrate the solution accuracy. The numerical solution is identified with markers while the exact solution is identified with a solid line.

Figure 3 shows the stress relaxation result for a single viscoelastic ply. Figures 4 and 5 illustrate a single viscoelastic ply subjected to a specified transient strain (the strains are plotted in Figures 4b and 5b respectively). Figure 4 shows the result as the ply is subjected to a 10% tensile periodic loading and Figure 5 illustrates the response under a 150% loading where the loading oscillates between tension and compression.

All Figures are scaled to show the normalized percentage of change to quickly assess the relative significance of the viscoelastic behavior.

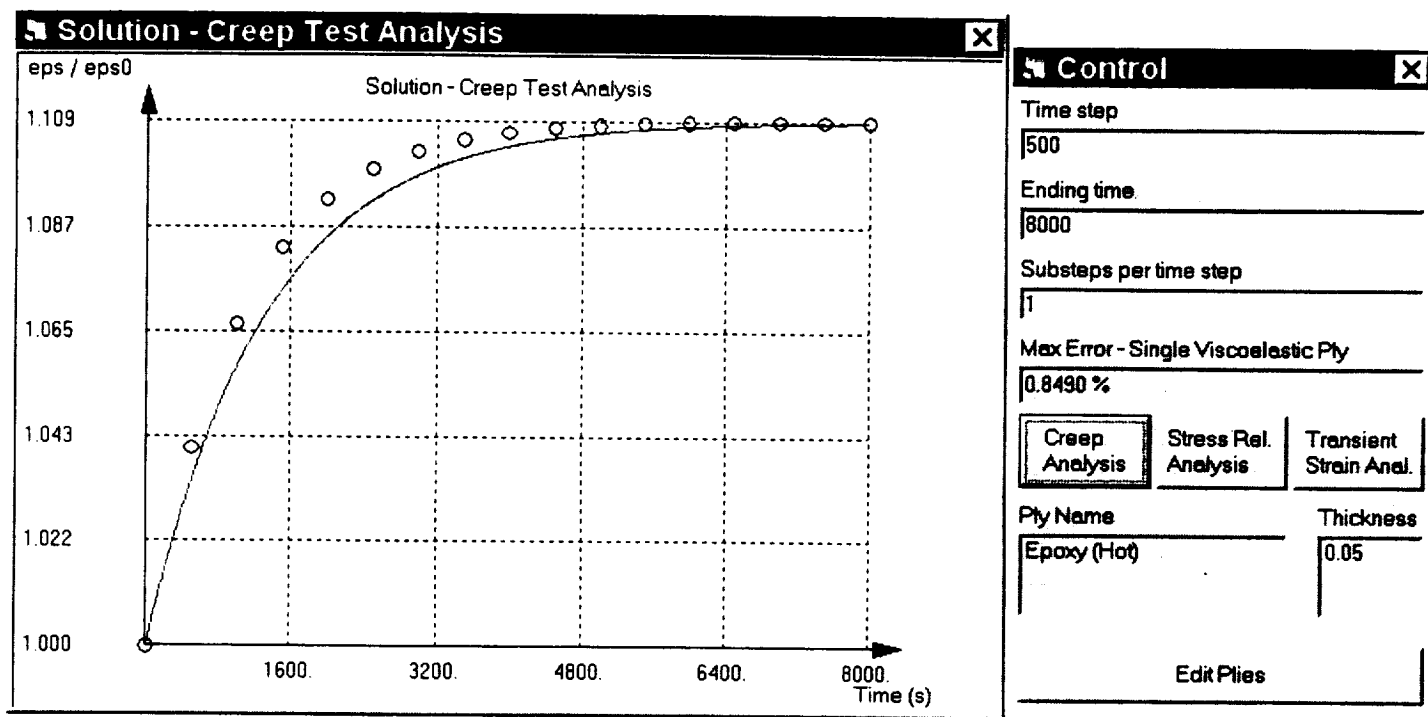


Figure 2a: Creep Analysis with 1 viscoelastic ply ( $\Delta t = 500$  s; 1 sub-iteration)

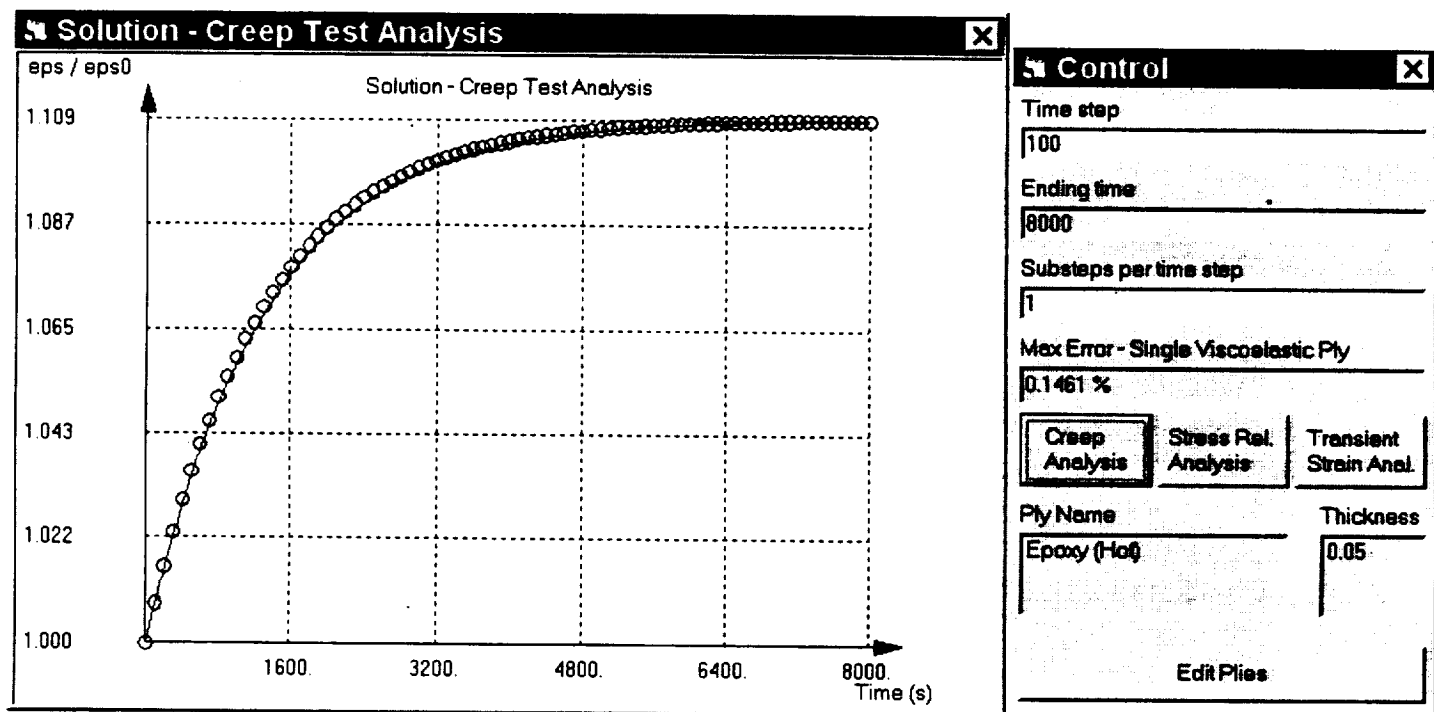


Figure 2b: Creep Analysis with 1 viscoelastic ply ( $\Delta t = 100$  s; 1 sub-iteration)

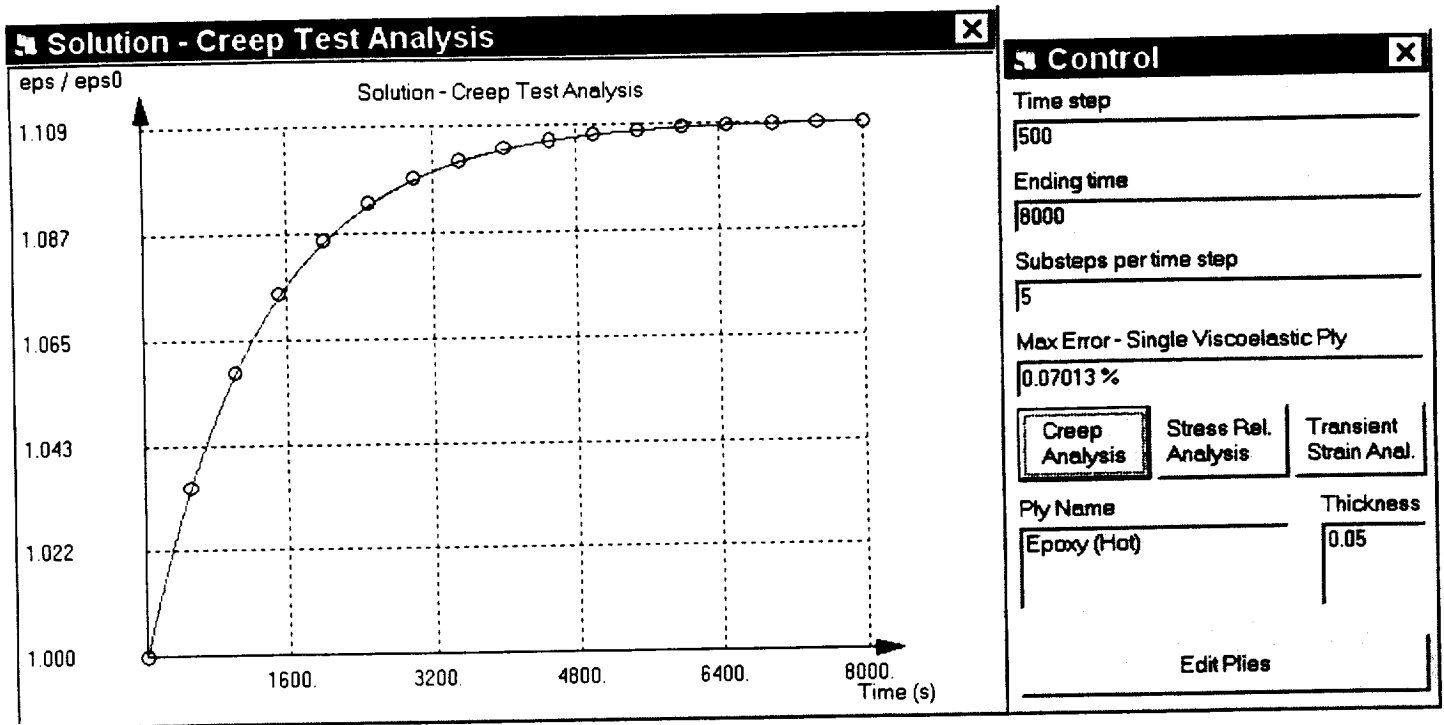


Figure 2c: Creep Analysis with 1 viscoelastic ply ( $\Delta t = 500$  s; 5 sub-iterations)

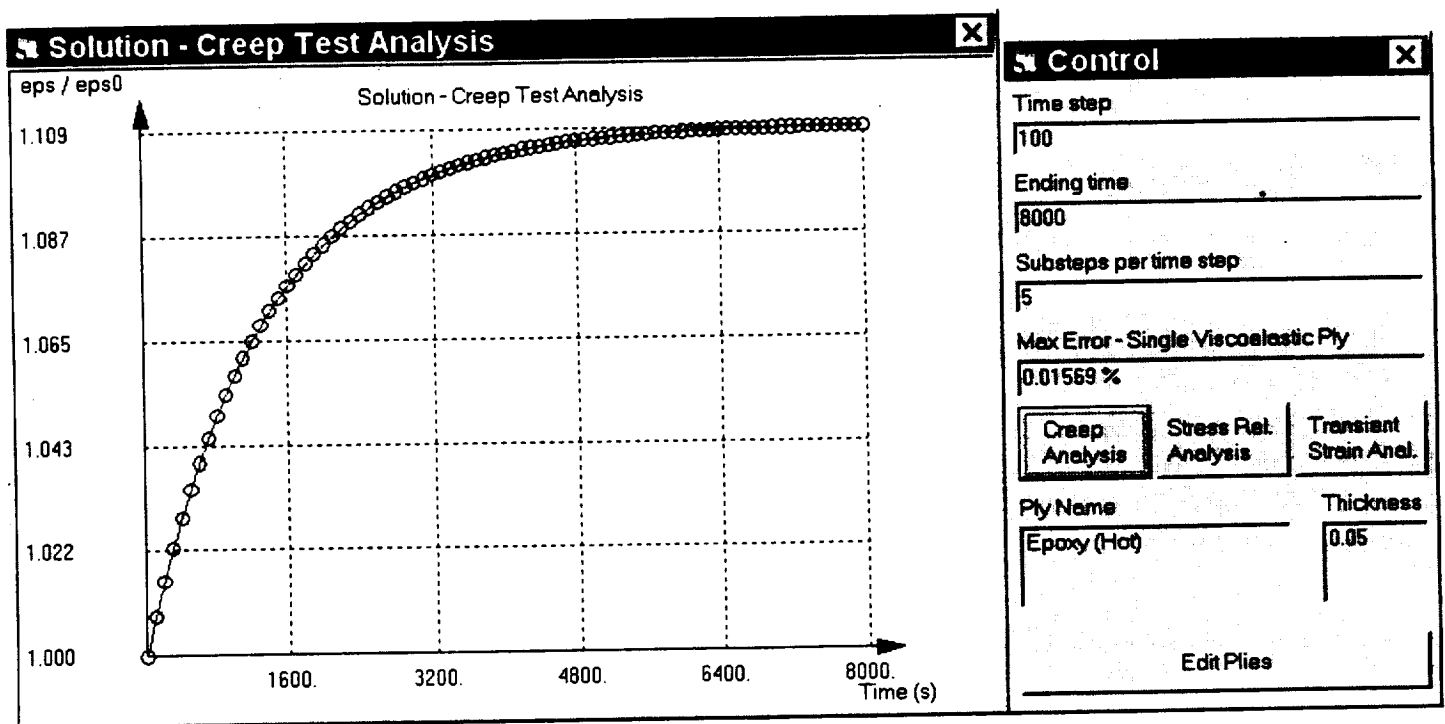


Figure 2d: Creep Analysis with 1 viscoelastic ply ( $\Delta t = 100$  s; 5 sub iterations)

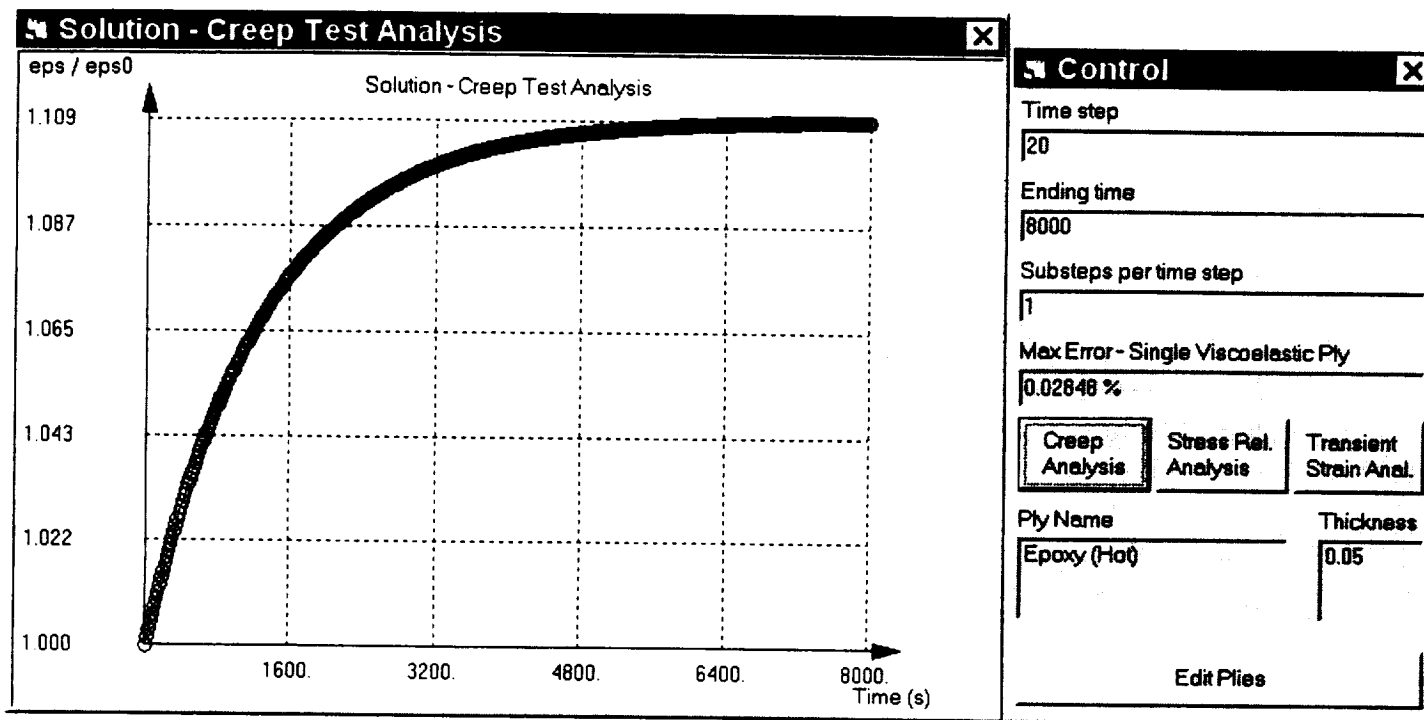


Figure 2e: Creep analysis with 1 viscoelastic ply ( $\Delta t = 20$  s; 1 sub iteration)

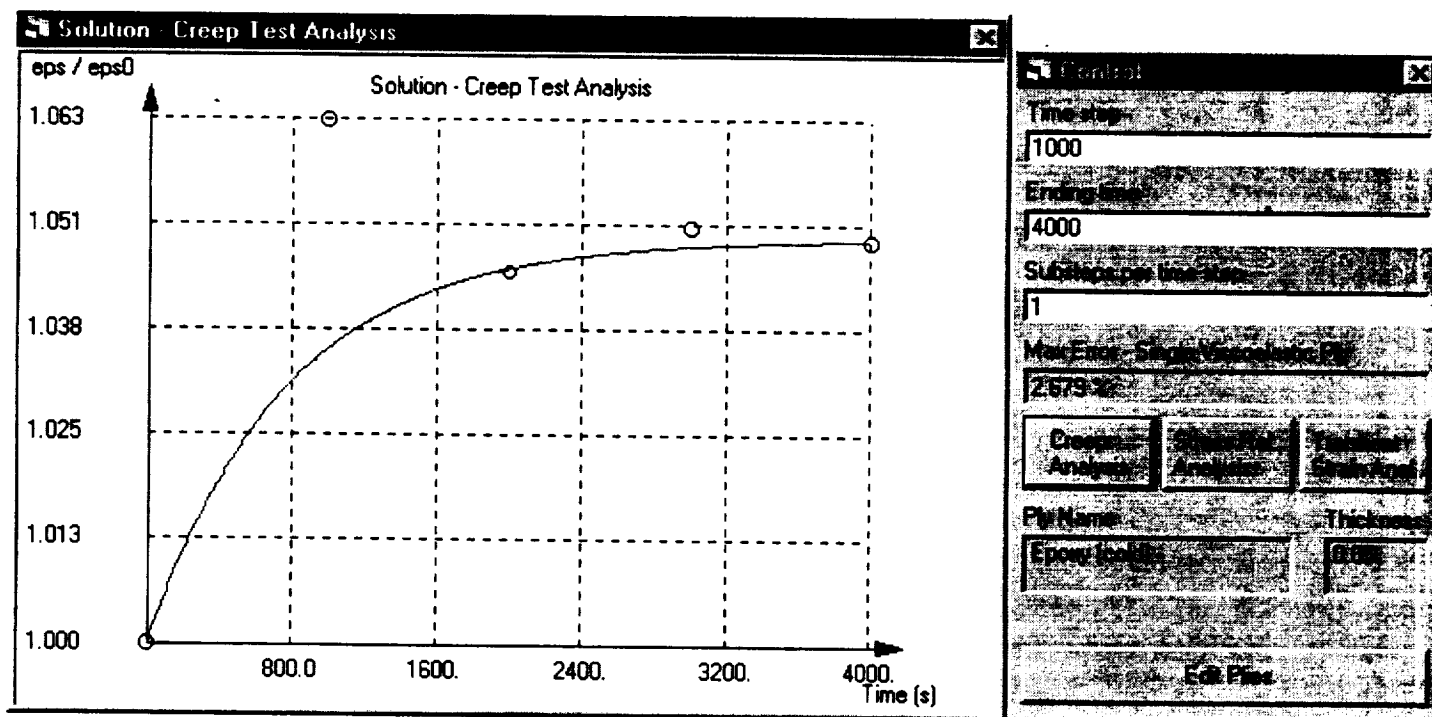


Figure 2f: Oscillations resulting from large timestep



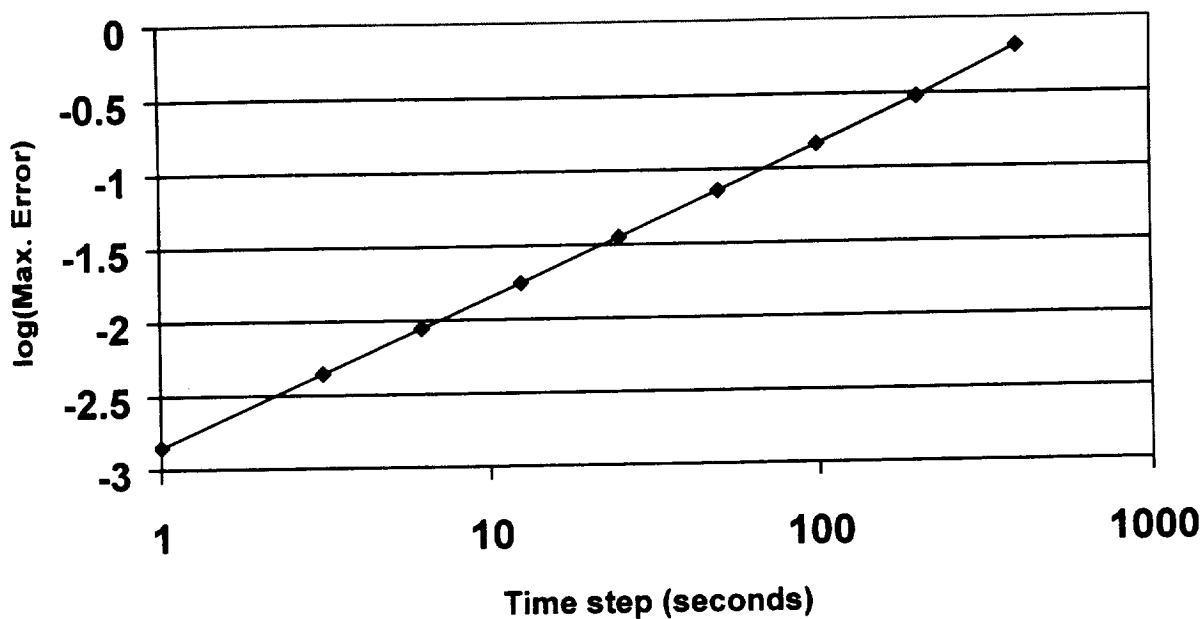


Figure 2g: Convergence of Creep result for a single viscoelastic ply

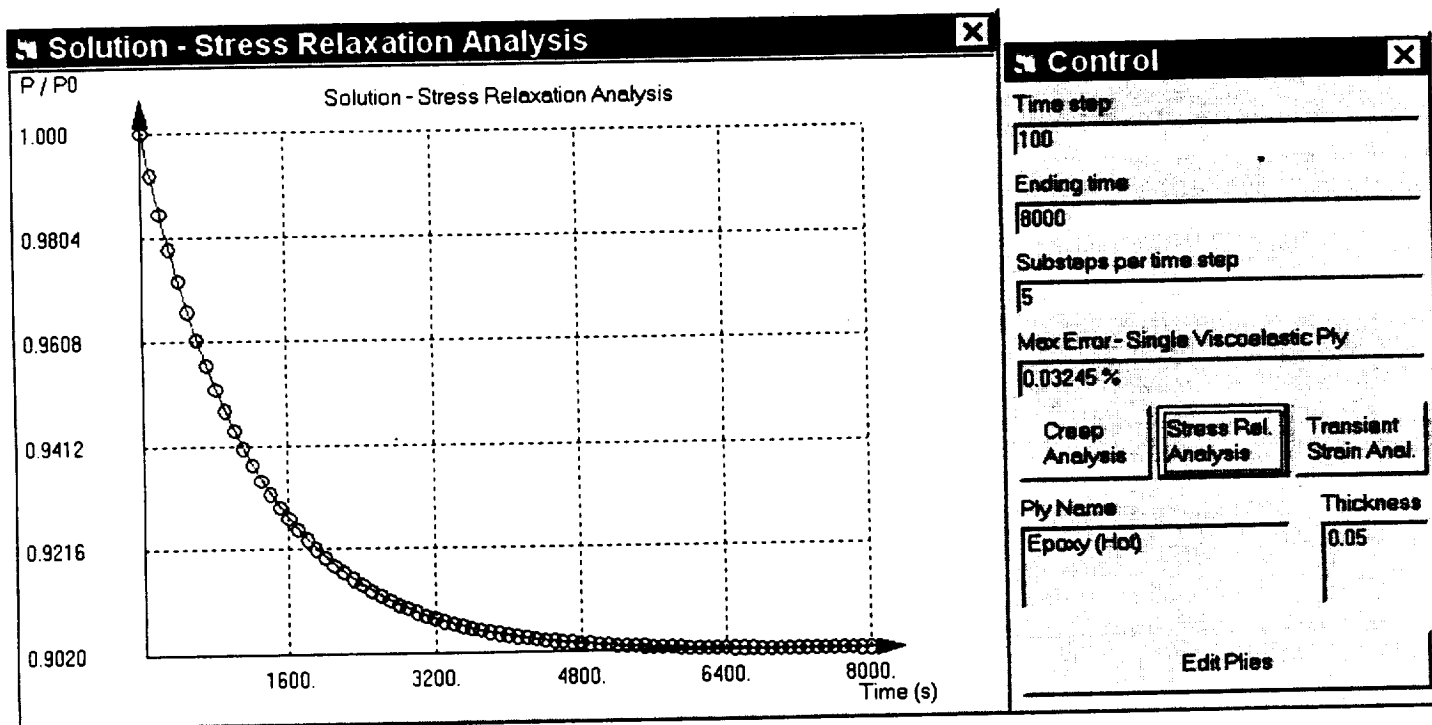
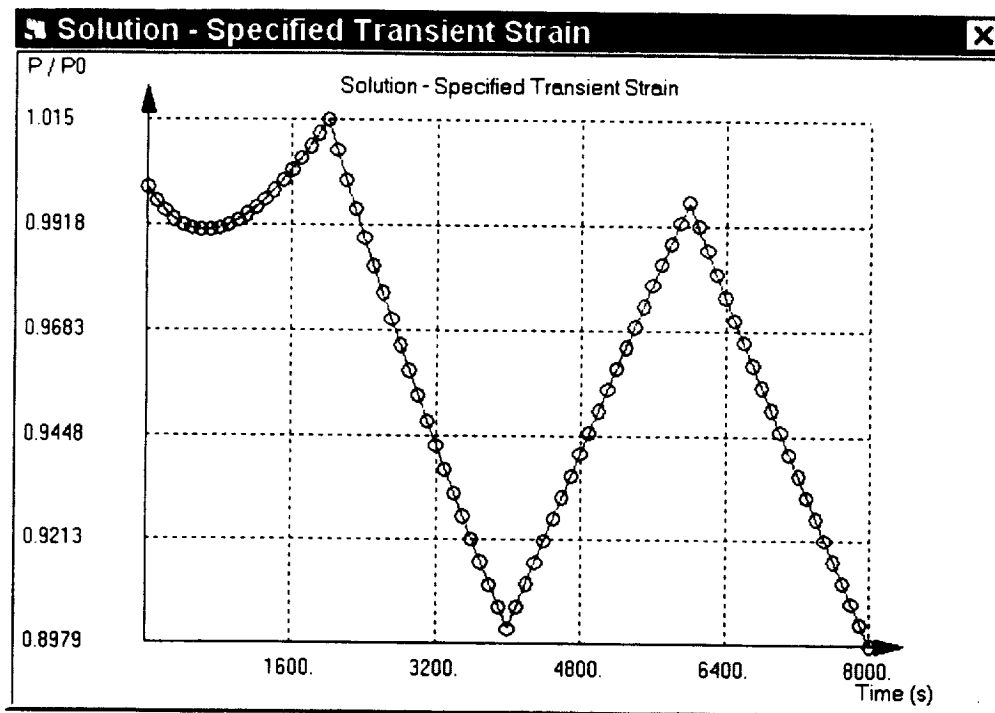


Figure 3: Stress relaxation result with 1 viscoelastic ply ( $\Delta t = 100$  s; 5 sub-iterations)



Control	
Time step	100
Ending time	8000
Substeps per time step	5
Max Error - Single Viscoelastic Ply	0.02818 %
Creep Analysis	Stress Rel. Analysis
<div>Transient Strain Anal.</div>	
Ply Name	Thickness
Epoxy (Hot)	0.05
<div>Edit Plies</div>	
Parameters	
Period of oscillations	4000
Magnitude (%)	10

Figure 4a: Specified transient strain solution ( $\Delta t = 100$  s; 5 sub-iterations)

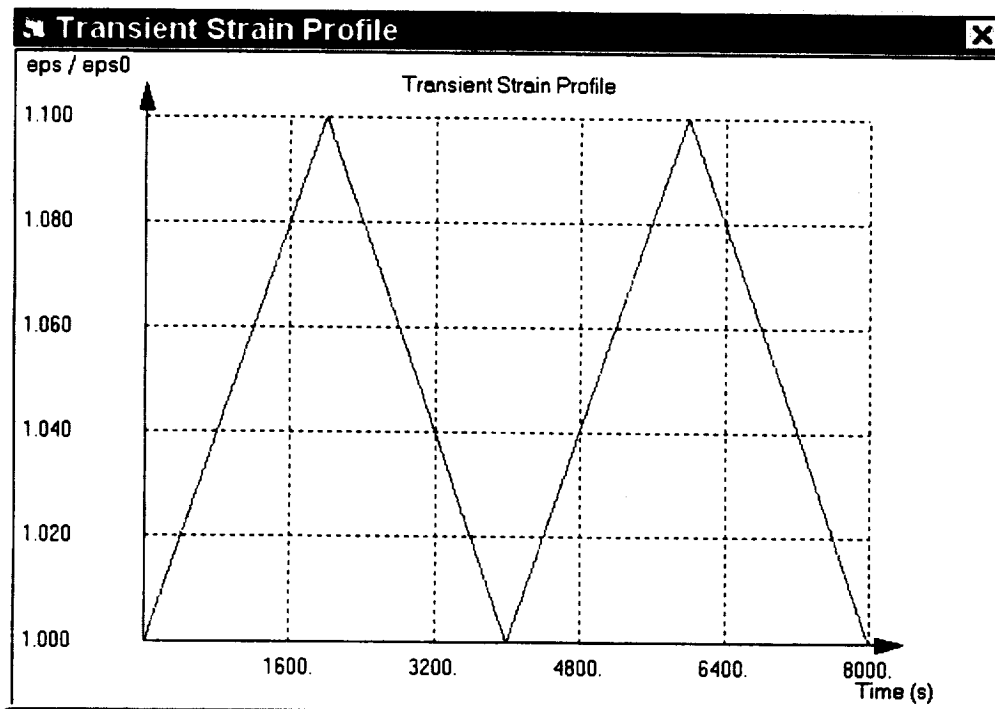
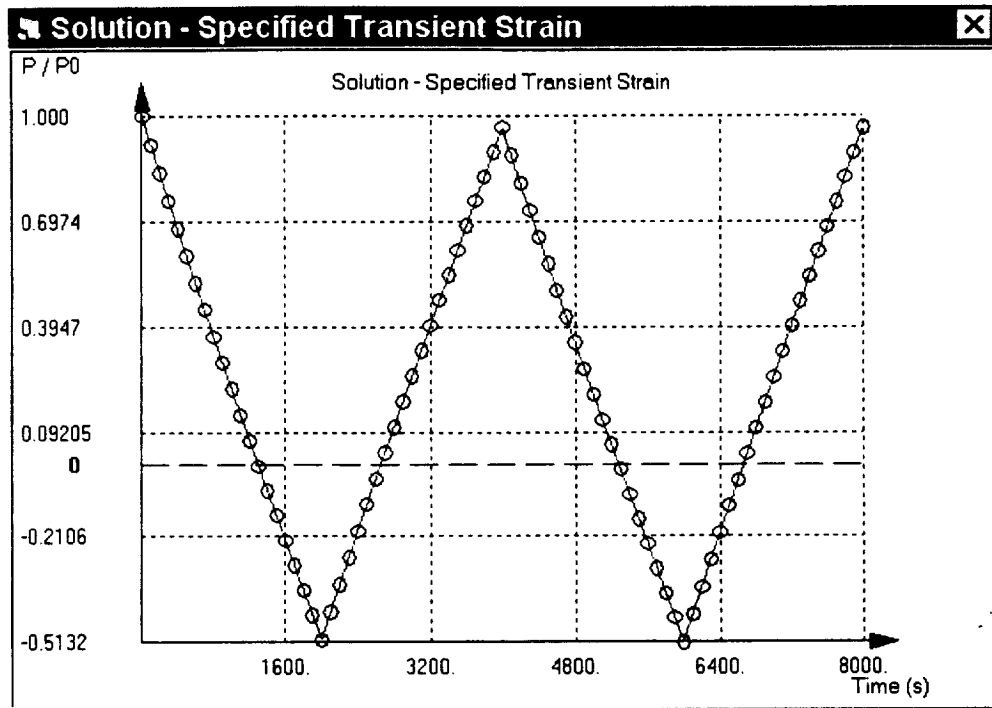


Figure 4b: Transient Strain Profile for Analysis



Control	
Time step	100
Ending time	8000
Substeps per time step	5
Max Error - Single Viscoelastic Ply	94.99 %
Creep Analysis	Stress Rel. Analysis
<b>Transient Strain Anal.</b>	
Ply Name	Thickness
Epoxy (Hot)	0.05
Edit Plies	
Parameters	
Period of oscillations	4000
Magnitude (%)	-150

Figure 5a: Specified transient strain solution (larger magnitude  $\Delta t = 100$  s; 5 sub-iterations)

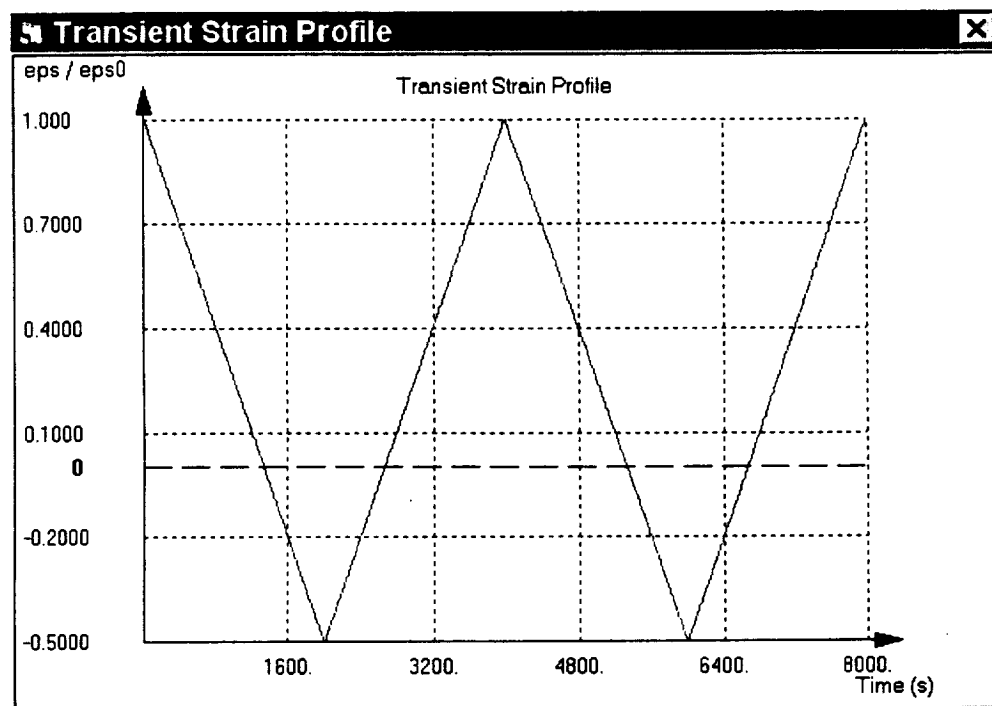


Figure 5b: Transient strain profile for analysis

Figures 6 and 7 illustrate creep results for the case of a viscoelastic and elastic ply and the case of 2 viscoelastic plies respectively. Because no closed form solution was obtained for the case of creep in two viscoelastic plies, the result was compared to a result obtained using a 4<sup>th</sup> order Runge-Kutta integration technique. As the 4<sup>th</sup> order technique is assumed to be much more accurate than Euler (which is 1<sup>st</sup> order accurate), the Runge-Kutta solution was assumed to be sufficient for comparison.

Figures 8 and 9 illustrate stress relaxation response for the case of a viscoelastic ply and elastic ply and the case of 2 viscoelastic plies respectively. Figures 10 and 11 demonstrate the same two cases under a specified transient strain loading profile.

Figures 12 - 15 illustrate some interesting features of the viscoelastic analyses involving multiple plies. In Figures 12 and 13 a laminate composed of one elastic and two viscoelastic plies is subjected to creep (constant stress) and stress relaxation (constant strain) loading respectively. In the case of creep, one can see that over time, the stress in the viscoelastic plies decreases as more and more of the stress is picked up by the elastic ply. Also, notice that the hot ply, which exhibits a stronger viscoelastic behavior picks up less of the total load than that of the cold viscoelastic ply. In Figure 13, the stress in the hot ply relaxes more than that of the cold ply, and of course, the stress in the elastic ply is constant. Figures 14 and 15 illustrate the same load cases with a laminate made up of two viscoelastic plies. Figure 14 is interesting because it shows that over time, the cold epoxy ply (which is less viscoelastic) exhibits an increase in stress while the hot epoxy experiences a decrease in stress. The net effect is a zero change in the laminate stress over time.

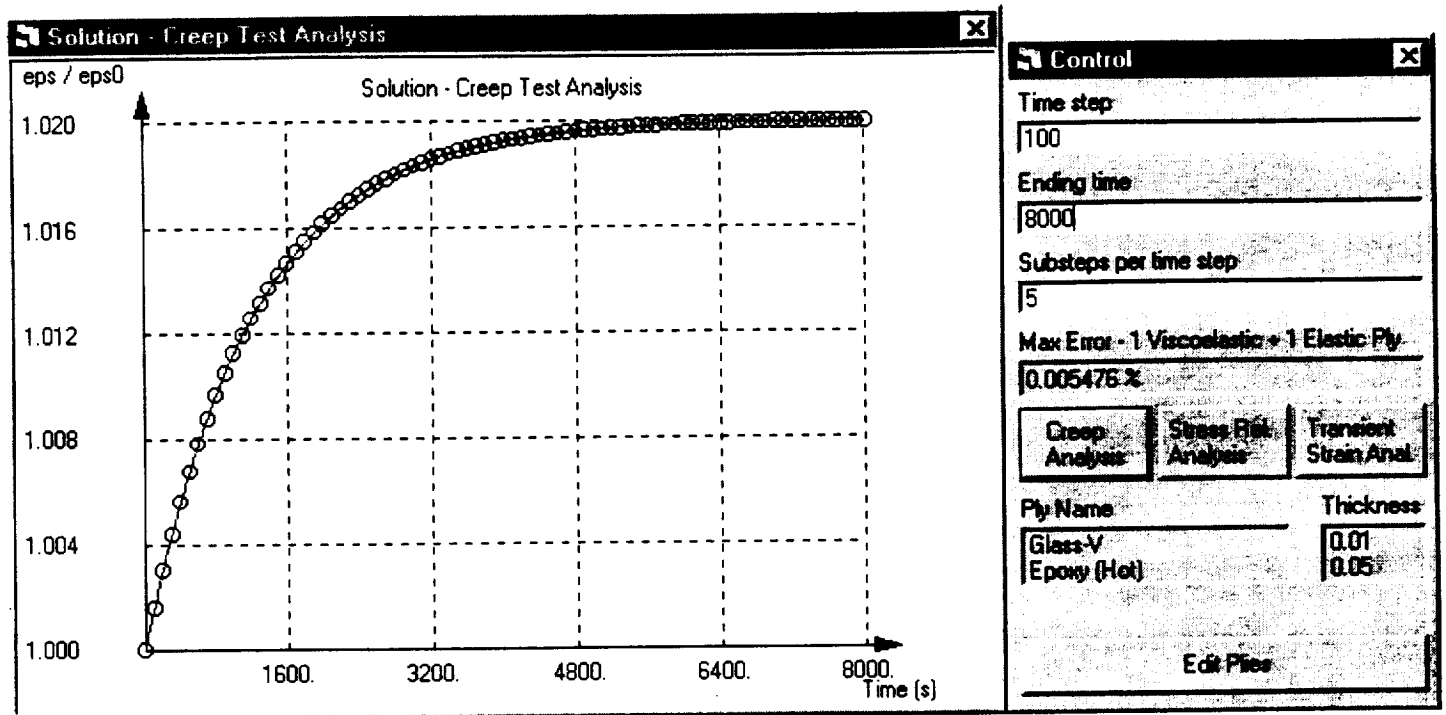


Figure 6: Creep Solution for one viscoelastic and one elastic ply ( $\Delta t = 100$  s; 5 sub-iterations)

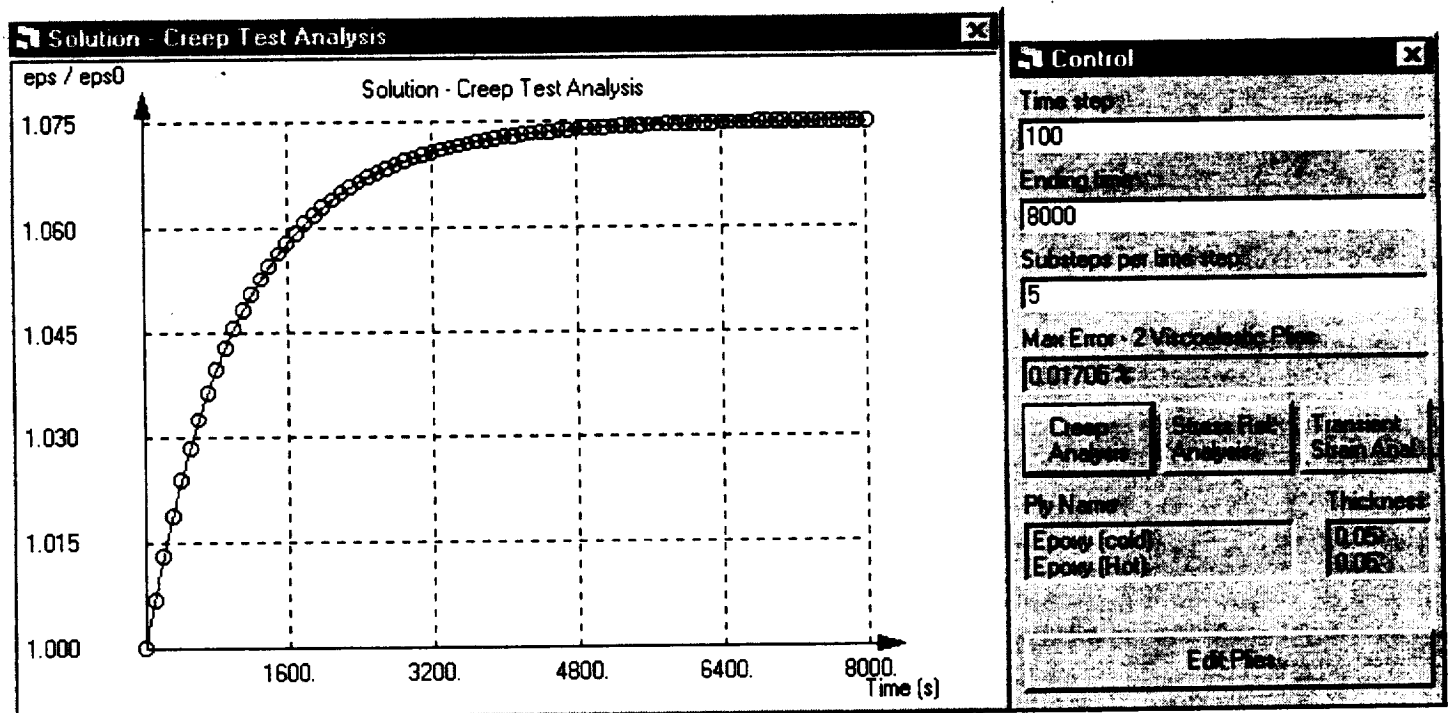


Figure 7: Creep result for two viscoelastic plies ( $\Delta t = 100$  s; 5 sub-iterations)

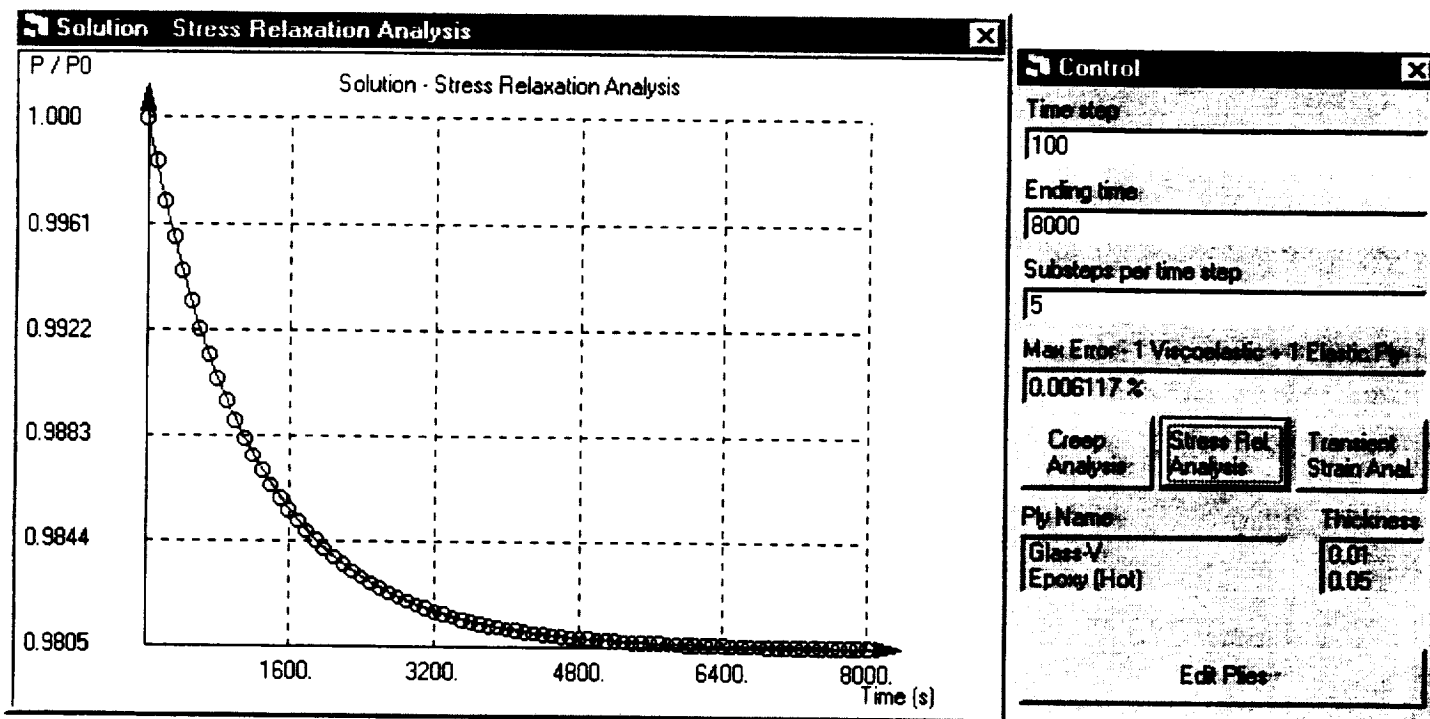


Figure 8: Stress relaxation result for one viscoelastic and one elastic ply ( $\Delta t = 100$  s; 5 sub-iterations)

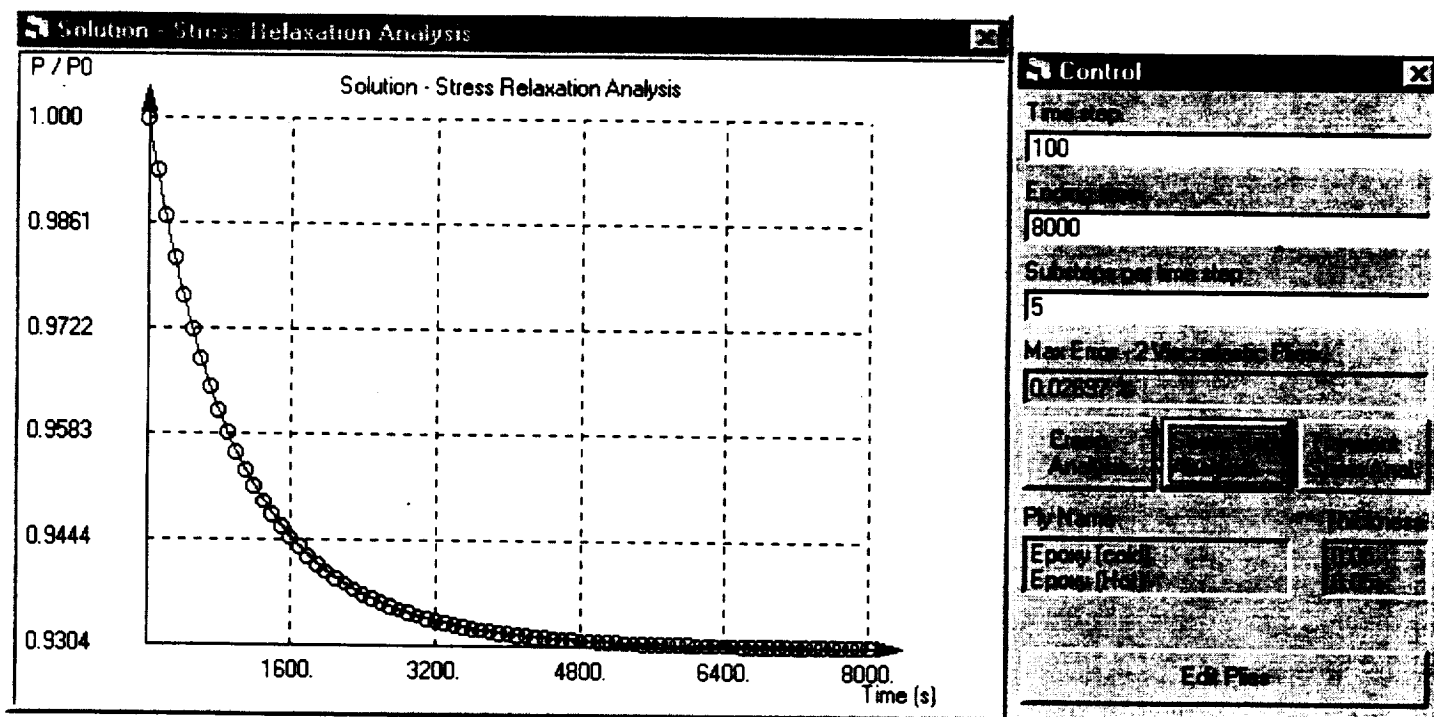
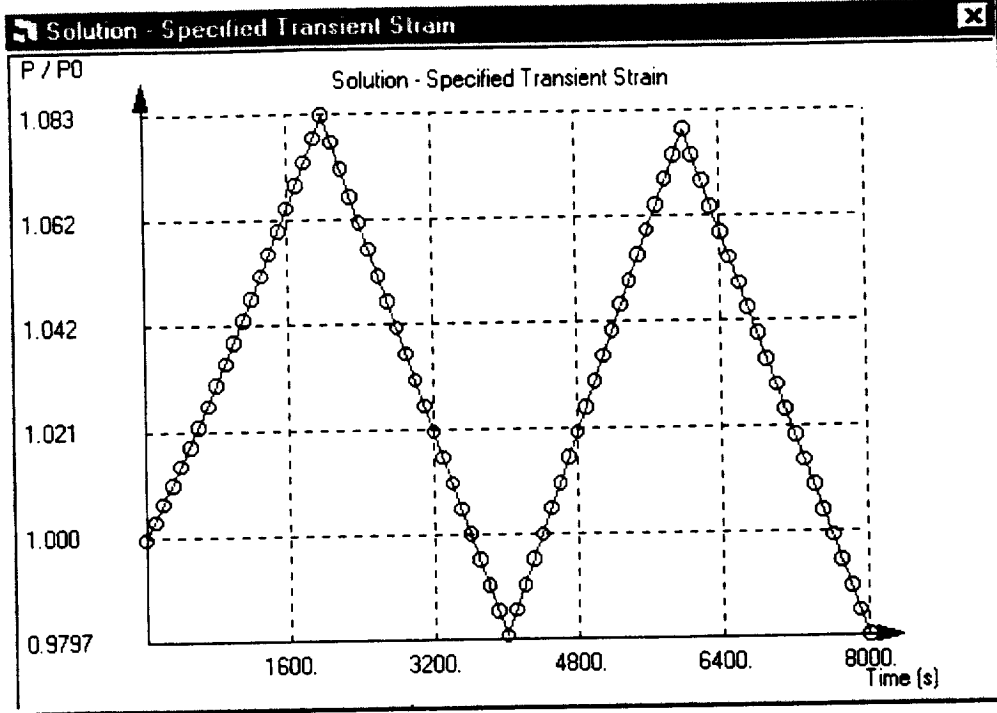
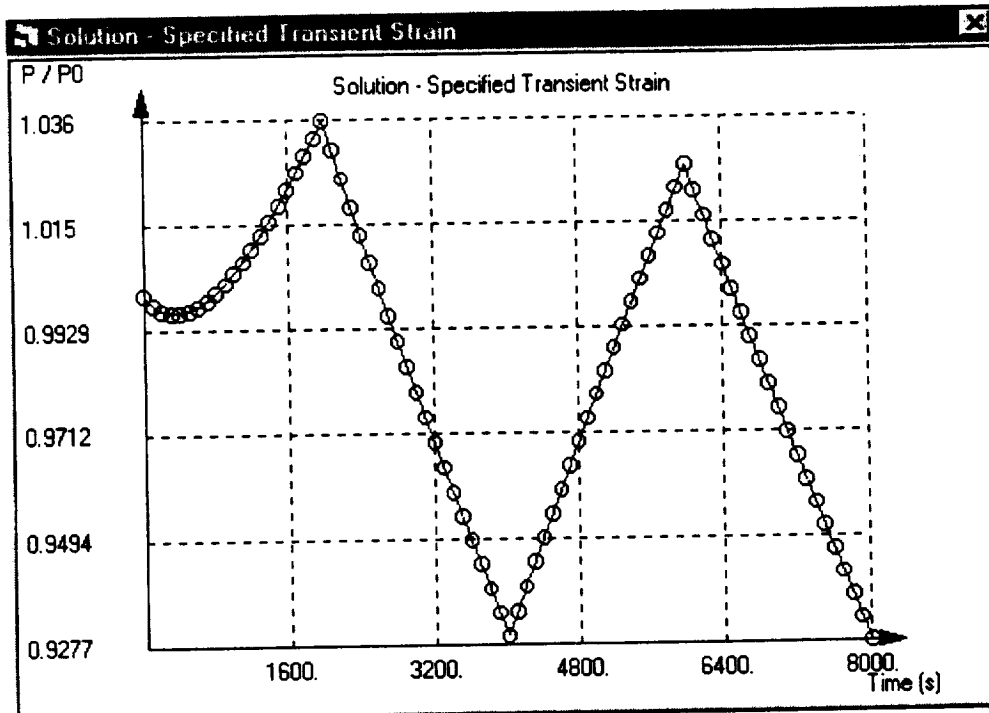


Figure 9: Stress relaxation result for two viscoelastic plies ( $\Delta t = 100$  s; 5 sub-iterations)



Control		
Time step		
100		
Ending time		
8000		
Substeps per time step		
5		
Max Error - 1 Viscoelastic + 1 Elastic Ply		
0.005144 %		
Creep Analysis	Stress Rel Analysis	Transient Strain Anal.
Ply Name		Thickness
Gless-V		0.01
Epoxy (Hot)		0.05
Edit Plys		
Parameters		
Period of oscillations		4000
Magnitude (%)		10

Figure 10: Specified strain result for one viscoelastic and one elastic ply ( $\Delta t = 100$  s; 5 sub-iterations)



Control		
Time step		
100		
Ending time		
8000		
Substeps per time step		
5		
Max Error - 2 Viscoelastic Plies		
0.01055 %		
Creep Analysis	Stress Rel Analysis	Transient Strain Anal.
Ply Name		Thickness
Epoxy (cold)		0.05
Epoxy (Hot)		0.05
Edit Plys		
Parameters		
Period of oscillations		4000
Magnitude (%)		10

Figure 11: Specified strain result for two viscoelastic plies ( $\Delta t = 100$  s; 5 sub-iterations)

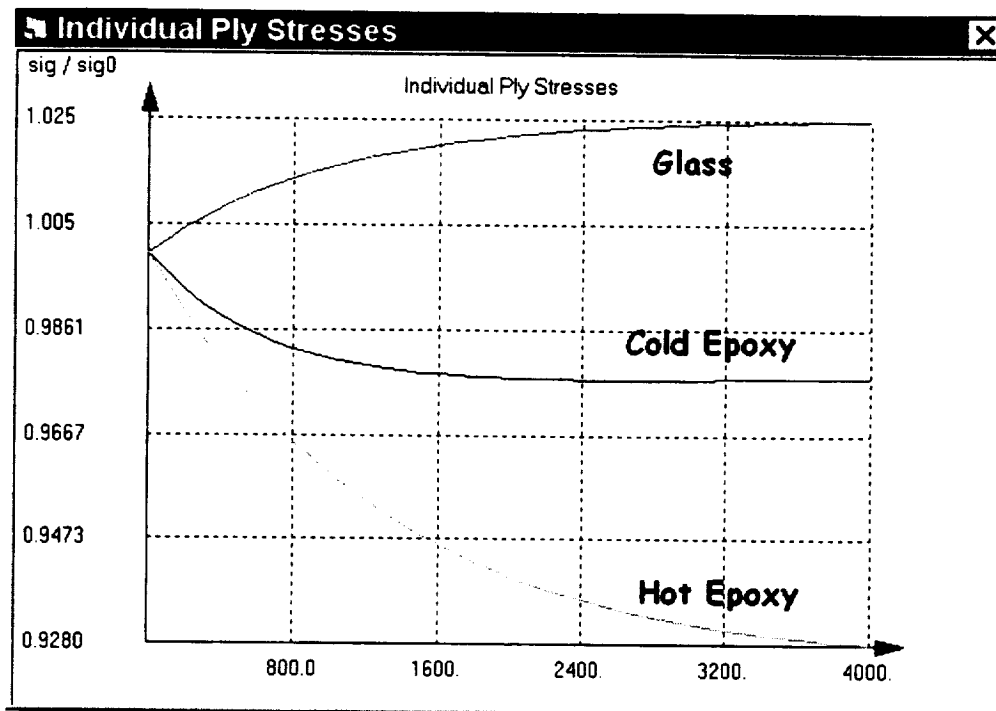


Figure 12: Individual ply stresses for two viscoelastic (epoxy) plies and one elastic ply (glass) undergoing creep

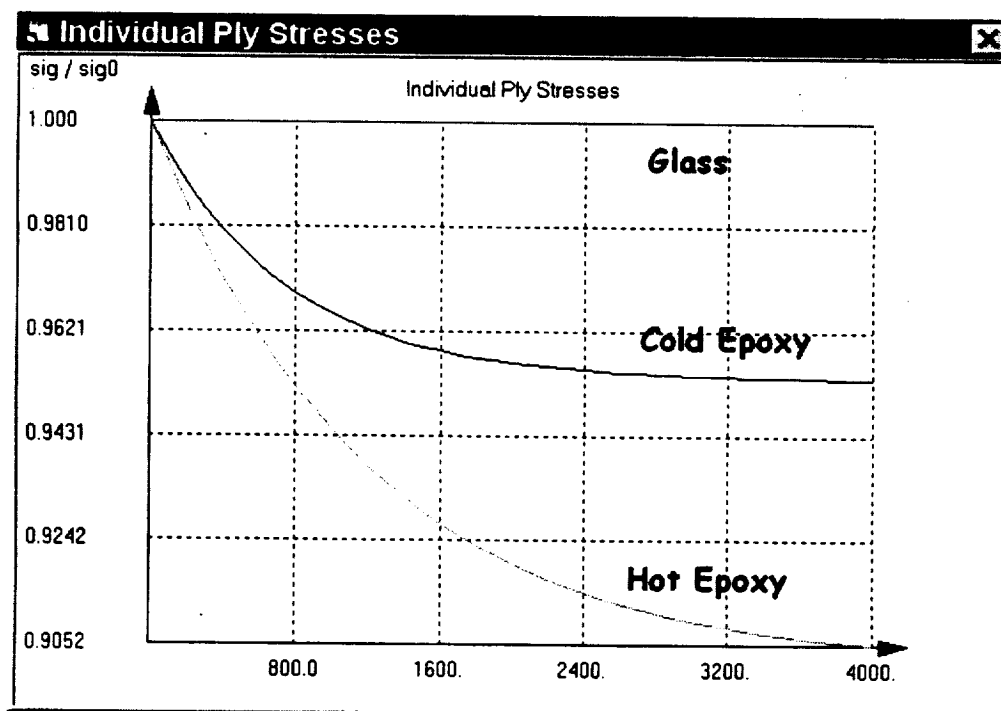


Figure 13: Individual ply stresses for two viscoelastic (epoxy) plies and one elastic ply (glass) undergoing stress relaxation



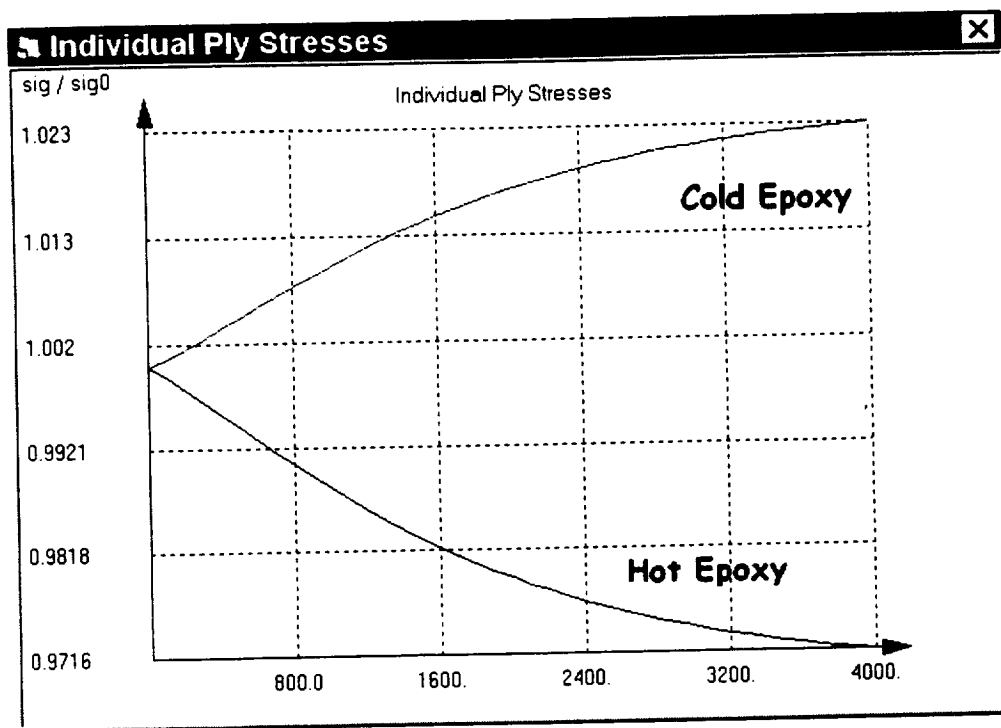


Figure 14: Individual ply stresses in two viscoelastic plies in undergoing creep

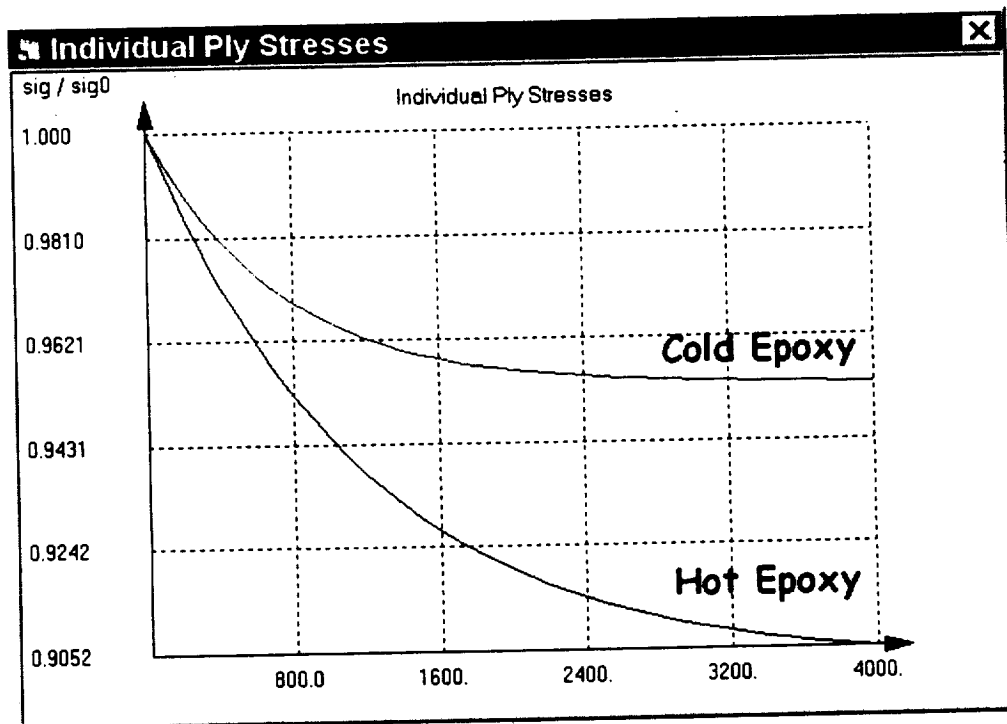


Figure 15: Individual ply stresses in two viscoelastic plies undergoing stress relaxation

## 6. Conclusions

Several conclusions can be drawn from the preceding results:

1. For the simplified problem of 1-D deformation in a laminate, the method of combining the responses of multiple plies to obtain a laminate response appears to be consistent. In other words, it converges on the correct result as the time step is reduced to zero.
2. The method is conditionally stable. The restriction on the time step is approximately equal to the minimum viscoelastic time constant,  $\tau$ , of the materials in the laminate.
3. The sub-iteration scheme, which allows several small time-steps on each individual ply before assembling those results into the laminate is effective in improving the overall accuracy while providing increased computational efficiency.

## 7. Future Work

The current work indicates that it is worthwhile to pursue the full panel formulation, eventually resulting in integration of viscoelastic plies into HyperSizer. There are two enhancements that are planned to be included in a separate "stand-alone" prototype code before attempting to integrate with HyperSizer. The first enhancement involves extending the present work to a full 3-D laminate including the effects of laminate curvature and Poisson effects. This algorithm will look very similar to that for a full panel as given in flowcharts 1 and 2. Second, it would be useful to integrate the Fortran subroutine, GMC\_VISC into the standalone code to determine the characteristics of using GMC in the algorithm before attempting to integrate with HyperSizer.

It also appears that the viscoelastic issues addressed and solved under the NRA funding will be directly applicable to developing a future viscoplastic laminate and panel capability.

The final phase of this effort is to integrate this algorithm into HyperSizer itself. The current implementation of the algorithm (with 3-D effects) can most likely be integrated directly into the code without GMC to model homogeneous viscoelastic materials (i.e. adhesive bond-coats, thermal barrier coatings, etc.) The inclusion of the viscoelastic algorithm with or without GMC will involve the inclusion of a time parameter and may be a substantial effort. At the present time, HyperSizer performs point analyses which are independent of time. A viscoelastic capability for polymer composite 'built-up' panels will be a substantial and valuable added capability in HyperSizer.

## References

- [1] Collier, C. Stiffness, Thermal Expansion, and Thermal Bending Formulation of Stiffened, Fiber-Reinforced Composite Panels, AIAA/ ASME/ ASCE/ AHS/ ACS 34th Structures, Dynamics, & Materials Conference, La Jolla, CA, April 19-22, 1993.
- [2] Golub, G., Ortega, J. Scientific Computing and Differential Equations: An Introduction to Numerical Methods, Academic Press, Inc. San Diego, CA, 1992.

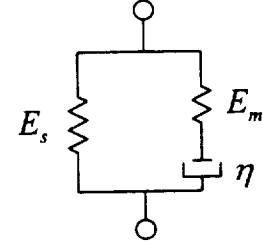


## Appendix A Analytical Results for Viscoelastic Formulation

### A.1 Results for a Single Viscoelastic Ply

#### A.1.1 Governing Equation

We are modeling a viscoelastic material using a modified Maxwell model as given in the Figure on the right. The quantity  $E_s$  represents the viscoelastic stiffness,  $E_m$  represents the viscoelastic stiffness and  $\eta$  represents the viscoelastic damping. The basic equation governing the overall response of this material is:



$$\dot{\epsilon} = \frac{1}{E_s} \dot{\sigma}_s = \frac{1}{E_m} \dot{\sigma}_m + \frac{\sigma_m}{\eta}$$

where  $\dot{\epsilon}$  is the strain rate and  $\sigma_m$  is the stress in the viscoelastic stiffness.

#### A.1.2 Stress Relaxation

For a constant strain value given by  $\epsilon_0$ , the stress is given by:

$$\sigma(t) = \epsilon_0 (E_s + E_m e^{-\gamma t})$$

where  $\gamma$  is the inverse of the material viscoelastic time constant and is given by:

$$\gamma = \frac{E_m}{\eta}$$

#### A.1.3 Creep

For a constant stress,  $\sigma_0$ , the strain is given by:

$$\epsilon(t) = \frac{\sigma_0}{E_s} \left( 1 - \frac{E_m}{E_s + E_m} e^{-\gamma t} \right)$$

where in this case, the time parameter is:

$$\gamma = \frac{E_m E_s}{(E_m + E_s) \eta}$$

#### A.1.4 Specified Transient Strain

Assuming a given linear strain profile given by:

$$\varepsilon(t) = mt + \varepsilon_0$$

The stress as a function of time can be expressed as:

$$\sigma(t) = \varepsilon(t)E_s + [\varepsilon_0 E_m e^{-\gamma t} + m\eta(1 - e^{-\gamma t})]$$

where  $\gamma = E_m/\eta$ .

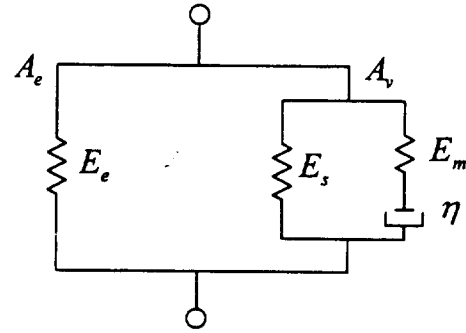
### A.2 Solutions for One Viscoelastic Ply and One Elastic Ply

#### A.2.1 Governing Equations

In this case, the system is modeled by the system shown at the right. The governing equations in each branch of the system are given by:

$$\dot{\varepsilon}_v = \frac{1}{E_s} \dot{\sigma}_s = \frac{1}{E_m} \dot{\sigma}_m + \frac{\sigma_m}{\eta}$$

$$\dot{\varepsilon}_e = \frac{1}{E_e} \dot{\sigma}_e$$



#### A.2.2 Stress Relaxation

Because the strain field is specified and always known, the viscoelastic stresses act independently of the elastic stresses, so the integrated stress is actually a linear combination of the stress in each branch of the model. Also, it makes more sense to speak of overall forces for the system since the two branches (elastic and viscoelastic) can have different areas (or thicknesses). The solution for force in this two ply laminate is given by:

$$P = P_e + P_v$$

$$P = \varepsilon_0 [A_e E_e + A_v (E_s + E_m e^{-\gamma t})]$$

where  $\gamma$  is the inverse of the time constant for the viscoelastic ply,  $\gamma = E_m/\eta$ .

### A.2.3 Creep

The solution for specified constant force,  $P_0$  is given by:

$$\varepsilon(t) = \frac{P_0}{A_e E_e + A_v E_s} \left[ \frac{A_e E_e + A_v E_s + A_v E_m (1 - e^{-\gamma t})}{A_e E_e + A_v (E_s + E_m)} \right]$$

where the time parameter is given by:

$$\gamma = \frac{E_m (A_e E_e + A_v E_s)}{\eta (A_e E_e + A_v (E_s + E_m))}$$

### A.2.4 Specified Transient Strain

As in the case of stress relaxation, because the strain is known, the elastic and viscoelastic sides act independently and the solution for force is given by:

$$P(t) = P_e + P_v$$

$$P(t) = A_e E_e \varepsilon(t) + A_v [E_s \varepsilon(t) + \varepsilon_0 E_m e^{-\gamma t} + m \eta (1 - e^{-\gamma t})]$$

where the time constant is taken from the viscoelastic branch as  $\gamma = E_m / \eta$ .

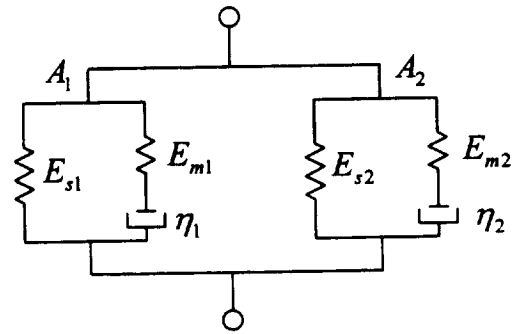
## A.3 Results for Two Viscoelastic Plies

### A.3.1 Governing Equations

In this case, the system is modeled by the system shown at the right. The governing equations in each branch of the system are given by:

$$\dot{\varepsilon}_{v1} = \frac{1}{E_{s1}} \dot{\sigma}_{s1} = \frac{1}{E_{m1}} \dot{\sigma}_{m1} + \frac{\sigma_{m1}}{\eta_1}$$

$$\dot{\varepsilon}_{v2} = \frac{1}{E_{s2}} \dot{\sigma}_{s2} = \frac{1}{E_{m2}} \dot{\sigma}_{m2} + \frac{\sigma_{m2}}{\eta_2}$$



### A.3.2 Stress Relaxation

As in the stress relaxation case outlined in A.2.2, the integrated stress for this two ply laminate is a linear combination of the independent stresses in each

branch of the model. The solution, cast in terms of the laminate internal force, is given by:

$$P(t) = P_1 + P_2$$

$$P(t) = \varepsilon_0 \left[ A_1 (E_{s1} + E_{m1} e^{-\gamma_1 t}) + A_2 (E_{s2} + E_{m2} e^{-\gamma_2 t}) \right]$$

where  $\gamma_1 = E_{m1}/\eta_1$  and  $\gamma_2 = E_{m2}/\eta_2$

### A.3.3 Creep

The case of creep in a laminate composed of two viscoelastic plies is the most complex solution presented here. The governing equations applied to the problem of creep reduce to:

$$\varepsilon(t) = \frac{P_0 - P_{m1} - P_{m2}}{A_1 E_{s1} + A_2 E_{s2}}$$

where the  $P_0$  is the complete force applied to the laminate and the forces in the viscoelastic branches ( $P_{m1}$  and  $P_{m2}$ ) of each ply are determined from the system of first order differential equations:

$$\begin{Bmatrix} \frac{dP_{m1}}{dt} \\ \frac{dP_{m2}}{dt} \end{Bmatrix} = \begin{bmatrix} \frac{-C_1}{(1-D_1D_2)} & \frac{D_1C_2}{(1-D_1D_2)} \\ \frac{D_2C_1}{(1-D_1D_2)} & \frac{-C_2}{(1-D_1D_2)} \end{bmatrix}$$

where:

$$C_1 = \frac{E_{m1}(E_{s1}A_1 + E_{s2}A_2)}{\eta_1(E_{s2}A_2 + A_1(E_{s1} + E_{m1}))} \quad C_2 = \frac{E_{m2}(E_{s1}A_1 + E_{s2}A_2)}{\eta_2(E_{s1}A_1 + A_2(E_{s2} + E_{m2}))}$$

$$D_1 = \frac{A_1E_{m1}}{E_{s2}A_2 + A_1(E_{s1} + E_{m1})} \quad D_2 = \frac{A_2E_{m2}}{E_{s1}A_1 + A_2(E_{s2} + E_{m2})}$$

This set of differential equations was not solved in closed form in the present work. Instead, a 4<sup>th</sup> order Runge-Kutta numerical integration scheme was used to find a solution which, while inexact, is much more accurate than the numerical solution resulting from the 1<sup>st</sup> order forward Euler algorithms presented here. The actual Runge-Kutta implementation is beyond the scope of this report, but can be found in any text on numerical methods. (e.g. Ref.[2])



#### A.3.4 Specified Transient Strain

As in the case of stress relaxation, the two branches of the model act independently and the solution is given by:

$$P(t) = P_1 + P_2$$
$$P(t) = A_1 \left[ E_{s1} \varepsilon(t) + \varepsilon_0 E_{m1} e^{-\gamma_1 t} + m \eta_1 (1 - e^{-\gamma_1 t}) \right] + A_2 \left[ E_{s2} \varepsilon(t) + \varepsilon_0 E_{m2} e^{-\gamma_2 t} + m \eta_2 (1 - e^{-\gamma_2 t}) \right]$$

where the time constants are  $\gamma_1 = E_{m1}/\eta_1$  and  $\gamma_2 = E_{m2}/\eta_2$ .

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